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Predicting Early Warning Signals of Financial Distress: Theory and Empirical Evidence

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Predicting Early Warning Signals of Financial Distress: Theory and Empirical Evidence*

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First version: March 1, 2013
This draft: October 31, 2017

Abstract

This study proposes a simple theoretical framework that allows for assessing financial distress up to five years in advance. We jointly model financial distress by using two of its key driving factors: declining cash-generating ability and insufficient liquidity reserves. The model is based on stochastic processes and incorporates firm-level and industry-sector developments. A large-scale empirical implementation for US-listed firms over the period of 1980-2010 shows important improvements in the discriminatory accuracy and demonstrates incremental information content beyond state-of-the-art accounting and market-based prediction models. Consequently, this study might provide important *ex ante* warning signals for investors, regulators and practitioners.

JEL classification: C63, C52, C53, G33, M41
Keywords: Financial distress prediction, probability of default, accounting information, stochastic processes, simulation

Data Availability: Data used in this study are available from public sources identified in the paper.

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1. INTRODUCTION

Since the early studies of Beaver (1966) and Altman (1968), the accounting literature has commonly assessed the economic health of companies by a wide variety of financial ratios and other financial information. Attributable to its academic and practical relevance, the state-of-the-art prediction models apply advanced statistical techniques that either employ *accounting-information* (Zmijewski 1984; Ohlson 1980; Beaver 1968b, 1968a; Altman 1968; Beaver 1966) or combine *stock market and accounting information* (Correia et al. 2012; Campbell et al. 2008; Bharath and Shumway 2008; Hillegeist et al. 2004; Vassalou and Xing 2004). However, applied statistical approaches, such as conditional probability models, multivariate discriminant analysis (MDA), proportional hazard models and similar technologies, have often been criticized for their inherent limitations and a lack of theoretical underpinning, as summarized by Beaver et al. (2011): "The literature on distress prediction has evolved without an explicit theory that specifies what financial statement ratios or how many ratios or what weighting approach will best allow assessment of the probability of distress. Instead, decisions regarding such issues have relied heavily upon intuition." Furthermore, the abovementioned approaches have a backward-looking perspective, are mainly static and single-period, and are pure statistical algorithms (Hillegeist et al. 2004; Mensah 1984; Vassalou and Xing 2004; Gentry et al. 1985). In addition, the exact default mechanism remains hidden as statistical models summarizes the default condition(s) in the estimated factor loadings of selected variables (Agarwal and Taffler 2008). Finally, the standard accounting-based models do not explicitly incorporate volatility measures such as asset volatility, which is a major driving force for bankruptcy and is thus prominently highlighted in many studies investigating the performance of market and accounting-based failure models (see e.g., Hillegeist et al. 2004; Shumway 2001; Das et al. 2009).

To remedy (at least some) of the abovementioned shortcomings, this study proposes a simple theoretical framework by combining research that forecasts financial statement information using stochastic processes for firm valuation[1] with the well-established bankruptcy prediction literature. In particular, we jointly model financial distress by two of its key driving factors: (1) declining cash-generating ability and (2) insufficient liquidity reserves. This model allows us to leave the static, backward-looking perspective of statistical models and instead extends the existing research by a

[1] (refernces include: Duffie and Lando 2001; Duffie et al. 2007; Favara et al. 2012; Garlappi et al. 2008; Garlappi and Yan 2011; Leland and Toft 1996; Pástor and Veronesi 2003, 2006; Schwartz and Moon 2000, 2001; Anderson and Carverhill 2012).

multi-period, theoretically grounded, forward looking approach. In particular, we implement two versions of our model: (1) an accounting-based model (henceforth *S-Prob*), and (2) an advanced model, which takes advantage of accounting and market information (henceforth *S-Prob*^{m}).

We benchmark these models empirically to current state-of-the-art accounting and market-based financial distress prediction models. In particular, we implement the following models using original and updated coefficients: Altman (1968) (*Z-Prob* and *Z-Prob*^{u}), Altman (1983) (*Z2-Prob*^{u}), Ohlson (1980) (*O-Prob and O-Prob*^{u}), Bharath and Shumway (2008) (*BhSh*^{u} *and BhSh-DD*^{u}), Campbell et al. (2008) (*C-Prob* and *C-Prob*^{u}), Correia et al. (2012) (*Beaver*^{u})[2] and the contingent claims framework by Crosbie and Bohn (2003) and Bharath and Shumway (2008) (*EDF*) on a broad sample of US-listed companies (i.e., NYSE, ARCA, AMEX, NASDAQ) over a thirty-year period 1980Q1-2010Q4.[3]

Following the existing literature, we draw the comparison along two dimensions to provide evidence on (1) discrimination ability and (2) information content.

Along the first dimension, discrimination ability, the results confirm that the proposed approach can improve the long-run discrimination power, measured in terms of the area under the receiver operating characteristic curve (AUROC). The baseline accounting-based model (*S-Prob*) is more accurate in distinguishing between non-delisting and delisting firms than the prominent *O-Score* or *Z-Score* models, even if applied with re-estimated coefficients (e.g., AUROC=0.8271 compared to AUROC=0.7828 and AUROC=0.7338 for the *O-Score* and *Z-Score* models). Regarding the market-driven approach *S-Prob*^{m}, the findings indicate an additional increase in accuracy compared to state-of-the-art market and mixed information models.

Along the second dimension, information content, the baseline accounting model (*S-Prob*) and the advanced market model (*S-Prob*^{m}) are incrementally significant compared to all benchmark models. Thus, the proposed measure carries information about future performance-related delistings not covered by state-of-the-art prediction models.

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[2] Although we cite the study by Correia et al. (2012), we label the model *Beaver*^{u}, because Correia et. al (2012) essentially build on the paper by Beaver et al. (2012).

[3] Details of the estimated benchmark models and the corresponding estimation results are provided in the Appendix.

We confirm prior results using several robustness checks and alternative research designs to further investigate the discrimination power to provide insights on a disaggregated level.

In sum, this study contributes to the debate of accounting-based vs. market-based financial distress prediction models in several ways.

First, this paper contributes to the literature by forecasting financial distress several years ahead in contrast to most studies focusing on failure prediction over the next period, e.g., one month or one year (see e.g., Chava and Jarrow 2004; Shumway 2001). While we acknowledge that some studies investigate longer forecast horizons (Campbell et al. 2008; Duffie et al. 2007; Reisz and Perlich 2007), this paper adds to this research strand by also investigating longer forecast horizons of up to five years. Campbell et al. (2008, p. 2900) describe the need for such long-forecast horizon models succinctly stating that distress prediction: "[…] may not be very useful information if it is relevant only in the extremely short run, just as it would not be useful to predict a heart attack by observing a person dropping to the floor clutching his chest."

Second, distress risk is modeled and triggered by two key components: (1) declining profitability and (2) insufficient liquidity reserves. Turning to the first component, we model the underlying sales and cost processes for each firm by mean-reverting stochastic processes explicitly incorporating accounting- based volatility measures. We employ industry-, time- and firm-specific parameters to capture a rich information set for each individual firm given that prior research has shown separately that all these drivers are important (see e.g., Chava and Jarrow 2004; Opler and Titman 1994). Our empirical analysis of the proposed approach shows significant cross-sectional and longitudinal improvements in the discriminatory power and demonstrate incremental information content beyond state-of-the-art financial distress measures.

Finally, our research design highlights that the equity-valuation literature inherently focusing on forecasting future firm performance can serve as a very promising device to model a firm's underlying distress process.

The rest of this study is structured as follows. Section 2 contains a brief overview of the financial distress prediction literature and introduces the benchmark models. Section 3 introduces the theoretical framework, while section 4 presents the data and the model implementation. Section 5 offers the results from the empirical tests. Finally, section 6 concludes and discusses limitations.

2. RELATED RESEARCH

The role of financial and non-financial information in modern credit risk theory dates back to the 1930s, and numerous metrics and statistical techniques have been proposed to predict the financial condition of firms. Although these models differ by the class of applied techniques, selected covariates, sample structures and the definition of failure, Ravi Kumar and Ravi (2007) report in their literature review that the majority of studies are based on accounting measures.[4] However, it is well known that relying solely on information from financial statements ignores substantial information (Ak et al. 2013; Beaver et al. 2012; Beaver et al. 2005). To overcome many of the limitations of accounting-based models and considering the heterogeneous nature of financial distress, the literature incorporates particularly information from the stock markets. Examples of prior research include the contingent claims framework by Black and Scholes (1973) and Merton (1974) and adapted, for example, by Sobehart et al. (2000), Vassalou and Xing (2004), Hillegeist et al. (2004), Bharath and Shumway (2008), and Das et al. (2009) or more recently by Correia et al. (2012) and Charitou et al. (2013). As shown by Shumway (2001), combining market-based with accounting-based covariates can increase the predictive ability of the traditional ad hoc accounting-based models. However, the empirical findings of Agarwal and Taffler (2008), Reisz and Perlich (2007), Das et al. (2009) and Xu and Zhang (2009) imply that the statistical performance between market-based and accounting-based prediction models remains at least competing. While market-based variables reflect additional, timelier information not captured by fundamental statements (Beaver et al. 2012), several studies suggest that accounting measures are incrementally informative to predict bankruptcy (Hillegeist et al. 2004). Xu and Zhang (2009) summarize that “the option pricing theory-based bankruptcy measure is more successful than the accounting variable-based measures alone, but it does not subsume the accounting measures.” Consistent with that, Batta and Wan (2014) show that accounting-based default models are less sensitive to stock market misvaluations than market-based default prediction

[4] For exhaustive literature reviews we refer to Dimitras et al. (1996); Scott (1981); Aziz and Dar (2006); Balcaen and Ooghe (2006); Ravi Kumar and Ravi (2007); Jackson and Wood (2013); Altman and Saunders (1998).

models. In this vein, Tian et al. (2015) emphasize the relative importance of 39 prevailing accounting and market-based variables to predict financial distress for different forecast horizons considering an advanced statistical selection technique.

The question then arises whether the benefits of market-based financial distress predictors could be incorporated for the vast majority of private firms in the United States having no shares listed on the stock exchanges?[5] Naturally, market-based models cannot be employed for private firms as they lack essential market data. At the same time, it is particularly important to evaluate default risk for non-listed firms, as they experience a high risk of default. Moreover, there is evidence that equity markets are not necessarily efficient. Prior studies such as Sloan (1996) find that the market does not accurately reflect all information in the financial statements. The findings of Tian et al. (2015) confirm that the prediction horizon affects the source of information included in financial distress prediction models. To account for potential market inefficiencies, particularly during times of volatile markets, as for example the dot-com bubble, the predictive accuracy might also benefit from putting more weight on accounting information rather than market data (Das et al. 2009).

Hence, as there is no clear (*ex ante*) superiority of one over the other source of information (i.e., accounting-based vs. market-based information) and the literature provides controversial arguments, we allow the stochastically driven framework to employ accounting and market measures. Being precise, we implement two versions of our model. First, a restricted version of the model using accounting-based information only, *S-Prob*, that can be compared to prior models purely based on accounting information and which has the potential to be employed for private firms. Second, we implement an advanced accounting and market-based parameterization, *S-Prob*^{m}, which can be compared to state-of-the-art models investigating samples of publicly listed firms.

Following Beaver (1966), a firm's financial distress risk depends on (1) the level of liquid assets, and (2) the cash flows from operations. Intuitively, firms with lower and more volatile cash flows and low liquid assets reserves to cover a potential shortfall should experience a higher probability to fail. It is

[5] The World Federation of Exchanges reports 5,215 listed companies on the NASDAQ/NYSE (www.world-exchanges.org as of May 2016). This accounts for less than 0.1 percent of US companies related to 23,735,915 nonemployer establishments or 5,726,160 enterprises reported by the Statistics of U.S. Businesses 2012 (SUBS) (http://www.census.gov/econ/susb). Similarly, Davis et al. (2007) report that private firms employ over two-thirds of the US workforce. A further discussion of the relative importance of private vs. public firms is offered by, e.g., Chen et al. (2011) and Hope et al. (2013).

well established that cash flows and operating performance are important variables in risk evaluation (Altman 1968; Beaver 1966; Emery and Cogger 1982; Jones and Hensher 2004; Shumway 2001). For the modeling of the operating cash flows, our approach is based on recent equity valuation models (Duffie and Lando 2001; Garlappi et al. 2008; Garlappi and Yan 2011; Pástor and Veronesi 2003, 2006; Schwartz and Moon 2000, 2001; Anderson and Carverhill 2012; Gryglewicz 2011). These studies model a firm's key profitability by stochastic processes to derive a value estimate for the firm similar to a discounted cash flow setting. The most recent models used in Garlappi and Yan (2011) and Pástor and Veronesi (2006) employ mean-reverting processes for revenues and earnings, respectively. For example, Garlappi and Yan (2011) build a simple equity valuation model to investigate the impact of shareholder recovery and the risk structure of equity; however, they obtain their probability of default estimates from Moody's KMV. Pástor and Veronesi (2006) seek to explain the high valuations of technology firms during the dot-com bubble rationally with respect to the prevailing uncertainty regarding the average profitability of these new business models. However, their model neglects the probability of default and does not include costs explicitly. Anderson and Carverhill (2012) simulate a well-designed structural equity valuation model with mean-reverting revenues. Their study is based on a single benchmark firm and focuses on the determinants of the optimal cash saving policy. Gryglewicz (2011) corporate finance theory studies the interaction between solvency, liquidity and dividend payout concerns in a dynamic equity valuation setting whereby cash flows are assumed to follow a Brownian motion. While all of these models are related to financial distress, neither study benchmarks its implied distress probabilities to prominent bankruptcy prediction models.

Hence, we focus on most commonly applied accounting and market-based specifications, which dominate the practice and research literature on financial distress prediction in terms of accuracy and explanation power. While there are several studies, for example Begley et al. (1996) and Hillegeist et al. (2004), that demonstrate that Ohlson' *O-Score* outperforms Altman's *Z-Score*, Agarwal and Taffler (2008) find that the *Z-Score* model outperforms other statistical models. Moreover, Altman and Saunders (1998) and Jackson and Wood (2013) report that multivariate discriminant analysis models and logit models are by far the most widely used statistical models. Other studies confirm that the two statistical models also perform well on more recent data (see e.g., Agarwal and Taffler 2008; Dichev 1998). In the recent literature, Shumway (2001) establishes the well-accepted and often used proportional hazard technique to allow for time-varying variables. While in many research studies the classical methods perform reasonably well, Shumway (2001), Chava and Jarrow (2004) and Bauer and Agarwal (2014) demonstrate a superior forecasting performance of proportional hazard models over logit, probit and multivariate discriminant analysis. Given its significant predictive and explanatory power the proportional hazard technique is the prevalent statistical technique applied to default prediction models and has been adopted in numerous studies such as Hillegeist et al. (2004), Campbell et al. (2008) and Correia et al. (2012), among others. Campbell et al. (2008) employ a proportional hazard model to unravel the distress puzzle, using monthly data from January 1963 through December 1998 and innovative covariates. For example, they scale accounting information by the market value of assets rather than the book value and integrate geometrically declining weights on lagged information. Their best failure model (No. 2) captures 31.2 percent of the variation in failure risks at a one-month prediction horizon. Correia et al. (2012) employ a large variety of prominent bankruptcy prediction models to link default probabilities to actual (forecasted) credit spread (changes). In terms of out-of-sample discrimination accuracy, their findings show that the proprietary EDF measure provided by Moody's/KMV outperforms corresponding bankruptcy prediction model (i.e., AUROC=0.9325 for a one-month prediction horizon). Consequently, more recent research by Crosbie and Bohn (2003); Hillegeist et al. (2004); Vassalou and Xing (2004), among others, advocate the contingent claims models based on the option pricing theory to predict immanent financial distress. As shown by information content analysis, the contingent claims model carries incremental information not captured by the *Z- and O-Score* specifications (Hillegeist et al. 2004). Bharath and Shumway (2008) criticize the underlying iterative approach of the contingent claims model as an insufficient statistic and provide a more accurate naïve version of a financial distress measure that mimics the methodology for the EDF measure.

While these benchmark specifications demonstrate improved out-of-sample discrimination and explanatory performance, the key intention of this study is not to build the best empirical forecasting model, but rather to develop an intuitive theoretical framework regarding the financial distress prediction process. The empirical implementation provides, nevertheless, compelling performance. In particular, we compare the stochastically driven model to accounting-based models by Altman (1968), Altman (1983) and Ohlson (1980) and to market-based and combined specifications provided by Bharath and Shumway (2008), Campbell et al. (2008) and Correia et al. (2012).

3. THEORETICAL FRAMEWORK

In this section, we provide a simple yet comprehensive non-linear model, where (1) the firm's operating performance and (2) the development of its liquid assets are the major driving forces for survival. The focus is on the firm's ability to generate sufficient cash inflows from its operations, which is critical in avoiding distress (Luoma and Laitinen 1991; Pompe and Bilderbeek 2005; Uhrig-Homburg 2005). Thus, section 3.1 and 3.2 develop the ingredients for the baseline *S-Prob*-model having only accounting information in mind. Section 3.3 extends the approach by showing how to additionally incorporate stock market information leading to our extended model *S-Prob*^{m}.

3.1. The Cash-generating Processes

First, with reference to the prior literature, we apply mean-reverting (Ornstein-Uhlenbeck) processes to formally derive a cash-generating process (Anderson and Carverhill 2012; Garlappi and Yan 2011; Klobucnik and Sievers 2013; Pástor and Veronesi 2006; Schwartz and Moon 2001). One major advantage of this approach is that it offers the opportunity to model the volatility of each process, because it is well established that volatility is a major driving force for financial distress (see Shumway (2001), Hillegeist et al. (2004) and more recently Correia et al. (2015)). Volatility measures, such as the assets volatility estimated from market data, play a major role in the contingent claims framework (e.g., Black and Scholes 1973 and Merton 1974) and capture systematic and idiosyncratic risk components (Kim 2012). Consequently, the decomposition of the cash-generating process into different stochastic processes captures the various sources of a firm's asset volatility. In particular, we incorporate four independent sources of volatility: accounting-volatilities, such as (1) sales volatility, (2) growth volatility, and (3) cost volatility, as well as market-volatilities, such as (4) stock return volatility.

Firms continuously experience more idiosyncratic volatility over the past decades in revenues, costs and cash flows given the inherent uncertainty of intangibles and a changing competitive environment associated with a decline in predictive ability (Srivastava 2014; Irvine and Pontiff 2009). Banker and Chen (2006), among others, find improvements in the predictive ability by disaggregating earnings into its major components. Accordingly, we begin by presenting the operating performance process by decomposed sales minus cost processes. This modeling approach is also well established in the literature (Anderson and Carverhill 2012; Garlappi and Yan 2011; Pástor and Veronesi 2006;

Schwartz and Moon 2001). It is a management accounting perspective where operating earnings are sales net of costs, which mimics the major cash flows of the firm. The sales dynamics (*S*) are given by the stochastic differential equation:

$$\frac{dS(t)}{S(t)} = g(t)dt + \sigma_S(t) \cdot dz_S(t) \tag{1}$$

where $g(t)$ is the sales growth rate, $\sigma_S(t)$ is the volatility of sales, and dz_s represents unanticipated changes in sales, which follow a Wiener process.

Second, consistent with previously research, we expect the profitability in a competitive environment to follow a mean-reverting process (Nissim and Penman 2001; Fama and French 2000), which means that the sales growth rate evolves according to the following:

$$dg(t) = \left[\kappa\left(\bar{g} - g(t)\right)\right]dt + \sigma_g(t) \cdot dz_g(t) \tag{2}$$

which best pictures the convergence to a long-run equilibrium growth rate $\bar{g}$ determined by market specifications as competition. The volatility of sales growth $\sigma_g(t)$ and the unanticipated changes in growth rates dz_g describe the unpredictable components (i.e., unsystematic noise) of the growth rate. The speed of convergence parameter κ captures the concept of adjusting to long-run equilibriums. Hence, the development of the sales is driven by innovations in the sales process and the underlying sales growth rate.

Third, in addition to sales, the cost component has to be modeled. Following the approach in Schwartz and Moon (2001) and Klobucnik and Sievers (2013), we abstract for simplicity from the separation of fixed and variable costs and model the selling, general and administrative expenses and the cost of goods sold together in the cost ratio c with a mean-reverting process:

$$dc(t) = \left[\kappa\left(\bar{c} - c(t)\right)\right]dt + \sigma_c(t) \cdot dz_c(t) \tag{3}$$

which also convergences to the long-run cost ratio $\bar{c}$ with the speed parameter κ and takes the volatility of the cost rate $\sigma_c(t)$ and the unanticipated changes in costs dz_c into account.

Regarding the volatility in the above processes, it is often assumed that maturing firms' businesses stabilize over time (Nissim and Penman 2001; Pástor and Veronesi 2003; Fama and French 2000). This means that the abnormal part for the volatility has faded away; thus, the risk in a firm's profitability reaches a steady, standard level. To illustrate the stabilization process, the three accounting volatilities are presented to proxy the idiosyncratic risk of sales, sales growth and cost that converge deterministically to a long-term normal level $\bar{\sigma}_S$, $\bar{\sigma}_g$ and $\bar{\sigma}_c$ with κ as the speed of convergence:

$$d\sigma_S(t) = \kappa \cdot \left[\overline{\sigma_S} - \sigma_S(t)\right] dt \tag{4}$$

$$d\sigma_g(t) = \kappa \cdot \left[\overline{\sigma_g} - \sigma_g(t)\right] dt \tag{5}$$

$$d\sigma_c(t) = \kappa \cdot \left[\overline{\sigma_c} - \sigma_c(t)\right] dt \tag{6}$$

By considering three sources of accounting volatility, the framework attempts to capture the different origins of volatility in a firm's operations.

From the processes above, the firm's earnings before interests, taxes, depreciation and amortization *EBITDA(t)* as basis for the operating cash flow is derived as follows:

$$EBITDA(t) = (1 - c(t)) \cdot S(t) \tag{7}$$

which is the gross profit margin $(1 - c(t))$ multiplied by sales. From the earnings before interests, taxes, depreciation and amortization the firm has to service its debt. Therefore, the interest positions also play an important role for the firm's liquidity. The net interest (interest income less interest expense) is modeled as follows:

$$Interest(t) = r_f \cdot cash(t) - r_{LTD} \cdot LTD \tag{8}$$

where *cash* is the interest earning cash position, and *LTD* is the interest-bearing long-term debt. *Cash* earns interest at the risk-free rate r_f, while the firm has to pay r_{LTD} for its long-term debt. We explicitly take the financial activities into account because recent research shows that firms significantly increased this position, e.g., to buffer the risk against future cash shortfalls (Bates et al.

2009; Lins et al. 2010; Sufi 2009; Anderson and Carverhill 2012; Opler et al. 1999; Acharya et al. 2012).

As result, the net cash flow from operations, *CFFO,* equals:

$$CFFO(t) = \begin{cases} (1 - tax) \cdot \left(EBITDA(t) + Interest(t) \right) & if \left(EBITDA(t) + Interest(t) \right) > 0 \\ \left(EBITDA(t) + Interest(t) \right) & if \left(EBITDA(t) + Interest(t) \right) \leq 0 \end{cases} \quad (9)$$

where *tax* is the corporate tax rate.

Being now equipped with the key *flow* item, describing how the firm generates or uses cash over time, the next section links *CFFO* to our major *stock* item, i.e., the firm's liquidity reserve.

3.2. The Change in Liquidity Reserve (LR)

To determine a measure of the firm's liquid assets as a buffer against unexpected cash outflows, we build on the bankruptcy literature. Davydenko (2012a) concludes in his research of liquidity and solvency measures in triggering financial distress that most of the financially distressed US companies in his sample between 1997-2010 are both illiquid (i.e., face a cash flow shortage) and economically insolvent (i.e., negative net value or over indebtedness). However, 10 percent in his sample of defaults appear to fail due to liquidity problems although having positive economic values. Hence, in addition to the high discrimination power of solvency measures and a firm's market capitalization, the importance of liquidity risk is increasing, particularly in the presence of financial frictions (i.e., financial constraints and covenants). Moreover, it has long been known (Merwin 1942; Smith and Winakor 1935) and is well established that the operating cash level is an important indicator in triggering a bankruptcy risk and is used, in terms of the net working capital position, in both the *Z-Score* and *O-Score* models. In his study, Altman (1968) finds that the working capital ratio is the only significant individual predictor for failure. In addition, there is evidence that the role of liquidity in the form of working capital has increased from the 1980s on (Begley et al. 1996). However, structural models, such as the contingent claims model, rely purely on the solvency structure and value-implied boundaries where default is triggered when market value falls below a certain (implied debt) threshold. Thus, liquidity risks are irrelevant in most of the structural models, as firms are assumed to overcome a cash shortage by raising new capital or selling assets (Leland and Toft 1996; Davydenko 2012b).[6]

Positive liquid reserves demonstrate the firm's ability to pay off its short-term and long-term obligations. As a measure of the available liquid funds, this indicates that suppliers and creditors could be satisfied from this amount. In contrast, Acharya et al. (2012) emphasize that the role of cash savings is associated with higher, not lower credit risk in the long term. Despite the importance of excess liquidity and positive economic value, research argues that (un)used lines of credit are an important instrumental component of corporates liquid funds. Therefore, we explicitly model the amount of unused line of credit available as additional liquidity buffer using the unique dataset of Sufi (2009).[7] By incorporating unused credit lines, we address firms operating with low or even negative working capital (e.g., Dell Inc., Boeing Co.) but high operating performance. This leads us to follow the findings of the bankruptcy literature and account for both, potential liquidity and solvency problems leading to financial distress and operationalize a proxy for the liquidity reserves as follows:

$$LR(t) = mean\left(S(t); L(t)\right) \tag{10}$$

where

$$S(t) = total\ assets + \ unused\ line\ of\ credits\ -\ total\ liabilities$$
$$L(t) = current\ assets\ +\ unused\ line\ of\ credits\ -\ current\ liabilities.$$

For the model, we assume that the liquid reserves evolve with the firm's cash flow from operations (see e.g., Emery and Cogger 1982; Anderson and Carverhill 2012). In case the reserves have been exhausted and drop below a certain level, the firm is technically in financial distress, analogous to

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6 Exceptions are structural models explicitly considering a cash flow shortage as a reason for default (e.g., Uhrig-Homburg 2005).

7 Sufi (2009) provides an indicator variable that equals one if a firm has a line of credit and zero otherwise for the period 1996-2003. Additionally, he reports the hand-collected data for the unused and used amount of credit lines disclosed in the 10-K filings. Specifically, we use this information to allocate an industry-specific median to approximate firms' unused line of credits according to the Fama-French (2015) 10-industry classification. Where hand-collected data are available, we allocate the precise amount of unused line of credit as reported in the 10-K filings. Negative values are set to zero. The results are robust to alternative allocations. There are 2,534 / 46,653 firms (firm quarter) observations with a predicted unused line of credit. We thank Amir Sufi for making his dataset publicly available: http://faculty.chicagobooth.edu/amir.sufi/chronology.html.

structural models. However, in contrast to the contingent claims framework, we trigger financial distress by a broader mechanism including liquidity measures and unused borrowing capacities. Moreover, the popular Black and Scholes (1973) and Merton (1974) model is based on a single aggregated stochastic process while the model proposed in this study employs different processes to model disaggregated risk dimensions. Overall, the larger the cash flows from operations and the liquidity reserves are, the lower the probability of financial distress predicted by the model is, which is in line with the results in Kahya (1997) and Anderson and Carverhill (2012).

Thus, the change in *LR* is driven by the cash flow from operations:

$$\Delta LR(t) = CFFO(t) \tag{11}$$

To arrive at the change in the *LR*, we assume no major cash flows from investing or financing activities for simplicity. Consequently, we abstract away from modeling investing in property, plant and equipment and there is no change in the capital structure.[8]

To implement the model, two adjustments are necessary. Compared with the daily frequency of market data in market-based structural models, the highest available frequency of accounting data is quarterly data. Hence, first, we discretize the stochastic processes to fit the quarterly frequency of accounting data. Second, due to the interaction of these processes, there is no closed form solution. Instead, we have to apply Monte-Carlo techniques to simulate the cash-generating sample paths. Given this study aims to assess the predictive power for financial distress for up to five years into the future, it is sufficient to simulate the *LR* paths up to the next twenty quarters. Similar to the structural models mentioned above, this study defines financial distress as state-dependent criterion, i.e., when the *LR* falls below a certain level *b*, the sample path is classified as failed and no longer simulated:

$$I_{\{LR(k)<b\}} = 1 \text{ if } LR(k) < b \mid LR(k-1) \geq \mathrm{b}, \text{ where } k \in (t+1, t+20) \tag{12}$$

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[8] However, by using selling, general and administrative expenses (item: xsgaq) and cost of goods sold (item: cogsq) from COMPUSTAT, we do consider research and development costs and amortization of tools and dies where the useful life is two years or less, which are part of investing activities. Moreover, we do take interest payments as part of the cash flow from operations into account, which can be regarded as financing activities in US GAAP. Including dividend payments as cash flow from financing activities and tax loss carry forward balances did not improve predictive power; therefore, we do not consider these items in this model.

Consequently, the empirical probability of financial distress is defined as the average inverse survival probability over the simulated sample period:

$$P(fd) = \frac{1}{T}\sum_{k=t+1}^{t+20}\left(\frac{\sum I_{\{LR(k)<b\}}}{N}\right) \tag{13}$$

where N=10,000 is the number of Monte Carlo simulations.

Regarding the financial distress boundary b, we need to answer when a firm enters the financial distress stage. For example, Reisz and Perlich (2007) and Davydenko (2012b) discuss findings for firm-specific boundaries. However, their implication refers to value-implied boundaries and not to liquidity specific boundaries. While the *LR* of solvent firms significantly differs by industry and period, the level for financially distressed firms naturally converges to zero. In line with the findings of prior literature (see e.g., Brockman and Turtle 2003; Reisz and Perlich 2007), we trigger a financially distress event at an early stage, i.e., before the financial obligations exceed the liquidity reserve. Therefore, we allow the barrier b to be positive and discuss further evidence in the empirical analysis (section 4.2). Next, we explain the extended parameterization of the *S-Prob*^{m} – measure, which allows market and accounting information.

3.3. A Market-based Parameterization

Quantifying the initial risk in the firm's operations by accounting volatilities ignores relevant non-financial-information contained in market prices. Thus, for our stochastically driven market approach, *S-Prob*^{m}, we modify the accounting-based volatilities to appreciate idiosyncratic risk structures contained in market-based volatility measures. To the extent that market-based information reflects significant more information than financial statements, we expect the modified volatilities (i.e., σ_s^m, σ_g^m and σ_c^m) to improve out-of-sample predictions. Accordingly, we determine modified volatility measures of the firm's risk in the cash-generating processes by systematically weighting accounting and market-based volatility metrics. The procedure outlined below indirectly quantifies the portion of market risks not explained by accounting data variability. Specifically, we estimate the following three rolling cross-sectional regression models (limited to 7 lagged firm quarters) with restrictions as follows:

$$\sigma_{acc,t} = \alpha + \beta_1 \sigma_{acc,t-1} + \beta_2 \sigma_{mar,t-1} + \varepsilon_t$$
$$s.t. \quad \alpha + \beta = 1 \qquad (14)$$
$$0 \le \alpha \le 1, 0 \le \beta \le 1$$

where $\sigma_{acc}(t)$ represents one of the three accounting-based volatilities ($\sigma_S(t), \sigma_g(t), \sigma_c(t)$). $\sigma_{mar}(t)$ is the firm's 3-month rolling standard deviation of daily stock returns centered around zero following the procedure outlined in Campbell et al. (2008). Basically, the modified accounting-volatilities for each firm at firm quarter t ($\sigma_s^m(t)$, $\sigma_g^m(t), \sigma_c^m(t)$) refer to the weighted values using the coefficients from the above regression models.[9]

Similar to the adjustment of the volatility measures, we consider the predictive value offered by the market capitalization to provide a more accurate liquid reserve measure. The underlying idea using the market capitalization as liquidity reserve is that it can be regarded as collateral in the case of bankruptcy. Following the corrective procedure in Campbell et al. (2008), we first modify our accounting-based proxy of the liquidity reserves and add 10 percent of the difference between the market capitalization and the liquidity reserves measured by financial statement information. Second, we replace our liquidity reserve proxy with the market capitalization in case the liquidity reserves exceed the market value of equity adjusted for minimum listing requirements.[10] This accounts in particular for liquidity effects not covered by financial reporting in the short run and shares trading at the edge of minimum listing requirements (Campbell et al. 2008; Macey et al. 2008).

$$LR^m(t) = \min\left(LR^{adj}(\mathrm{t}); ME(t)\right) \qquad (15)$$

where

9 Details are given in Appendix.

10 The majority of stock exchanges require that companies to meet minimum criteria depending on the listing standards. Commonly are minimum criteria referring to fundamentals (e.g., positive and continually performance measures) or market capitalization (e.g., a specific minimum bid price or average market capitalization). Given the variety of listing standards in our sample and the prior findings about the significance of the \$1.00-rule of Macey et al. (2008) and the insignificance of fundamental listing requirements by Rhee and Wu (2012) and Chen and Schoderbek (1999), we employ a most conservative market-based rule and adjust the market capitalization for the maximum of either the \$1.00-rule or a \$5 million of market capitalization. Hence, the minimum listing requirement is calculated: Max(no. of shares; \$5 mio.). Similar, Campbell et al. (2008) adjust the price variable and truncate price above \$15.

$$LR^{adj}(t) = LR(t) + 0.1\left(\mathrm{ME(t)} - LR(t)\right)$$
$$ME(t) = market\ capitalization\text{ -}minmum\ listing\ requirement$$

The next section describes the data and discusses the empirical estimation results compared to the introduced benchmark models.

4. DATA AND MODEL IMPLEMENTATION

4.1. The Data

Our sample comprises quarterly accounting data from 1980Q1 to 2010Q4 for the US market. Thereby, the results are comparable with prior research as the majority of studies are conducted for the time period 1980-2003 as presented in Ravi Kumar and Ravi (2007). At the same time, they are more up to date because the sample additionally covers the recent financial crisis. There are three reasons for starting in 1980 in the literature. First, Dichev (1998) demonstrates that the COMPUSTAT sample contains substantially more firms after 1980 and that business failures have significantly increased after 1980. Second, the Bankruptcy Reform Act of 1978 changed the institutional setting and economic risk factors of financially distressed firms (Hackbarth et al. 2015; Hillegeist et al. 2004). Third, since both the *Z-Score* and the *O-Score* models were developed before 1980, the out-of-sample perspective is guaranteed. While most studies use annual accounting data, this study offers results on a more favorable quarterly frequency. Chava and Jarrow (2004) and Campbell et al. (2008) demonstrate that the higher frequency of quarterly data, which contain more timely information, can improve forecasting abilities of the models. We exclude (1) firms with SIC-codes 6,000 to 6,999 (financial firms) and (2) firm quarter observations where essential accounting information were unavailable. The equity price data and delisting codes are taken from CRSP daily and monthly databases for NYSE/AMEX/NASDAQ/ARCA common stocks (share codes 10/11). To include delisting firm quarter observations not in the date range of the CRSP/COMPUSTAT merged databases (CCM), we follow the methodology of Beaver et al. (2007) for an extended merge procedure.[11] The accounting data is lagged by two months to ensure that the data are observable prior to a delisting, i.e., it is assumed that financial statements are available by the end of the second month after the firm's fiscal quarter-end (Campbell et al. 2008; Correia et al. 2012).

Thus, there remains a large and anonymous dataset of 330,274 firm quarter observations for 10,747 non-financial firms from CRSP/COMPUSTAT with non-missing information. While many bankruptcy studies work with small datasets, large samples as in Chava and Jarrow (2004) offer more convincing results.

[11] We would like to thank Richard Price for providing the associated SAS program on his website: http://richardp.bus.usu.edu/research.

This study adopts exchange-delisting codes from CRSP, which are common in the literature and more reliable given the underlying delisting regulations and standards existing for US stock exchanges. In addition, CRSP delisting information offers explicit dates and reasons for a delisting. Regarding the economic impact after the delisting date, Macey et al. (2008) report that stocks delisted from the NYSE and subsequently traded on the so-called “Pink-Sheets” nearly double their mean volatility of closing prices, triple their average percentage spreads and simultaneously lose approximately 50 percent of their share price and daily trading volume. There are different sets of delisting codes related to financial distress/business failure in the literature depending on the broadness of the definition. Most studies use the definition of performance-related delistings (see for instance: Campbell et al. 2011; Caskey et al. 2012; Dichev 1998; Reisz and Perlich 2007; Shumway 2001). Performance delistings are associated with negative changes for the firm and cover the CRSP delisting codes 400 and 550-585. In addition to bankruptcy, performance delistings additionally include, among others, insufficient capital, market capitalization or market-makers and non-payment of exchange fees (Campbell et al. 2008; Dichev 1998). Similar to Xu and Zhang (2009), we consider all these cases as financially distressed. In addition, delisting codes provide two further advantages. First, certain difficulties to predict cases such as strategic reasons (relief of debt) or unexpected events (natural disasters) receive less weight allowing us to focus on a severe financial weakness. Second, a broader definition ensures to capture cases where firms have to counter serious financial problems, but could avoid filing for bankruptcy or willfully delay their filings (Hilscher and Wilson 2016). In general, our broad financial distress definition may capture more firms than relying only on the legal definition of bankruptcy where the default occurs after a firm is already in a financial distress situation.[12]

By using this broad measure of distress, we obtain 41,506 firm quarter observations and 3,483 firms that were delisted for performance-related reasons. This yields a cumulative firm delisting rate of 32%, which is substantially higher than for example the 14% in Reisz and Perlich (2007), who work with a smaller sample and a shorter sample period. Table 1 offers an overview regarding the reasons and distributions of delistings and clearly shows that performance delistings can be associated with financial distress or bankruptcy.

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[12] Consistent with prior literature, we winsorize independent variables at the 1st and 99th percentile of their pooled distributions to prevent extreme values from driving the results. The results are robust to alternative specifications (e.g., winsorizing by fiscal quarters to account for possible time trends).

[Please insert Table 1 about here]

4.2. Parameter Estimation based on Accounting Information

The baseline accounting-based model has in total 18 parameters, which stem from the firms' balance sheets and income statements. However, the employed information set is the same as for the *Z-Score* and *O-Score* and other accounting-based models.

The estimation of the model initial input parameters is presented in Table 2.

[Please insert Table 2 about here]

The parameter values are based on intuitive and straightforward estimations using firm- and industry-specific moments of the preceding seven quarters to control for seasonal effects. Where there are fewer than seven preceding quarters available, we restrict the estimation to the available information set (with a minimum of four quarters) to keep as many observations as possible in the sample. This is another advantage of the model compared to the statistical models, which demand more data to be initialized as argued above. Hence, our model does not need an initial calibration period or a holdout sample approach to test its performance (Hillegeist et al. 2004). Moreover, this model is less sensitive to structural breaks, as it depends on (short-term) accounting data and estimates parameters firm-specifically.

In the following, a few key parameters are described, while we refer the reader to Table 2 for the remaining estimates. First, the initialization starts with a set of variables provided by the companies' financial statements: sales (S_0), variable cost as fraction of revenues (c_0), interest-bearing debt (LTD_0), cash position ($Cash_0$) and the liquidity reserves (LR_0). The initial sales growth rate is determined by the intercept of a recursive least squares rolling firm-by-firm regression model (with seven lagged firm quarters) that relates the current quarterly sales growth rates to the one quarter lagged sales growth rate. The initial sales volatility ($\sigma_{S,0}$) and volatility of the sales growth rate ($\sigma_{g,0}$) for each firm is defined by the firm quarter-specific mean standard deviation of the residuals, i.e., the root mean squared error, and the standard deviation of the ordinary least squares slope coefficient. The intuition is that an increase of sales and sales growth volatility decreases the persistence of sales and sales predictability. In the same vein, we develop a proxy for the initial volatility of costs ($\sigma_{c,0}$) based on the mean standard deviation of the residuals from a regression of the variable cost rate on the one quarter lagged variable cost rate (with seven lagged firm quarters). This approach is inspired by Francis et al. (2005) developing an accrual quality measure.

Srivastava (2014) documents noticeable decreasing cost intensities and an increase in revenue and cost volatilities associated with cohorts of newly listed firms from 1970-2009. Hence, for the long-term parameter of costs ($\bar{c}$) we estimate a rolling cross-sectional median over the preceding seven quarters to capture systematic changes over time. To account for time-trends in the underlying volatility measures, we set both the long-term volatility of sales ($\bar{\sigma}_S$) and the long-term volatility of costs ($\bar{\sigma}_c$) to their rolling cross-sectional median. To eliminate unsystematic noise in the sales growth rate processes we set both the long-term sales growth rate ($\bar{g}$) and the long-term volatility of the sales growth rate ($\bar{\sigma}_g$) to zero. In this case, the sales growth processes stabilize over time, and the cash-generating process is only driven by innovations in sales and costs processes to converge to a "normal" growth pattern in the long run.

The debt interest rate (r_{LTD}) is defined as the interest expense divided by the book value of current and long-term debt as in Francis et al. (2005). The corporate tax rate is defined as the top federal statutory corporate tax rate (see, e.g., Nissim and Penman (2001)). For the risk-free rate (r_f), we choose the short-term risk-free rate represented by the 3-Month US-Treasury Bill rate.[13]

The estimation of the speed of convergence parameter (κ) deserves more explanation. In the economics literature, the standard approach to estimate convergences is to use the absolute convergence regression (Rogoff 1996). Similar to Altomonte and Pennings (2008), we use the following regression to estimate the convergence: [14]

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[13] The risk-free rate and industry-classification portfolios were taken from Kenneth French's library.

[14] Additionally, we use the approach of Klobucnik and Sievers (2013) to calculate the speed of convergence, i.e. the kappas. Solving $\sum_{i=t-5}^{t-8} \frac{saleq_i - saleq_{i-1}}{saleq_{i-1}} = \left(\sum_{i=t-1}^{t-4} \frac{saleq_i - saleq_{i-1}}{saleq_{i-1}}\right) \cdot e^{-4 \cdot \hat{\kappa}}$ for *kappa* yields an alternative estimator, which is then pooled to medians for the same industry (three digit SIC codes). The estimated default probabilities and the other findings remain qualitatively the same. However, the approach in this study is well established in the economics literature and therefore preferable.

$$\ln\left(\frac{g_{t,i}}{g_{t-1,i}}\right)=\alpha_i+\beta_i\cdot g_{t-1,i}+\varepsilon_{t,i} \tag{16}$$

where $g_{t,i}$ is the corresponding sales growth rate for time t and firm i, and β_i yields the firm-specific convergence. These firm-specific values are then pooled for each industry (Fama-French 48-Industry classification) to define the median as industry-specific convergence parameters for two main reasons. First, prior studies (e.g., Chava and Jarrow 2004; Hillegeist et al. 2004) clearly demonstrate that industry effects are important for bankruptcy prediction (because of different levels of competition, among others). Second, industry-specific parameters provide more stable estimates and neutralize individual outliers in the large dataset.

A final key parameter in our stochastic model is the boundary condition b, i.e., the level of the liquidity reserves to classify a firm as financially distressed. Empirically, the level when a firm enters a financially distress stage is associated with economic determinants. We find reliable evidence in time-series analysis that (1) the distress barrier related to liquid assets is time dependent and frequently decreases during times of worsening economic conditions, and (2) controlling for firm-specific, industry-specific or stock exchange-specific determinants do not improve the discrimination accuracy. Prior research mainly focuses on the market-value of assets to specify a certain default boundary condition (Davydenko 2012b; Reisz and Perlich 2007). For example, Reisz and Perlich (2007) estimate the mean (median) implied barrier level at 30.53% (27.58%) of the market value of assets (i.e., non-zero). In the same vein, the results by Davydenko (2012b) show a corresponding mean (median) barrier level that equals 66.0% (61.6%) of the face value of debt. Further, Chen and Schoderbek (1999) findings suggest that the AMEX continued listing requirements referring to an accounting-based threshold are incapable to trigger delisting procedures given that 45.7% of firms did not violate the listing requirements before their delisting. Regarding firm-specific boundaries, Davydenko (2012b) document a slight reduction in discrimination accuracy compared to fixed boundary levels. Theoretically, the financially distressed firms in our sample enter a downward spiral long before the liquidity reserve is exhausted and drops to zero. Keeping these different results in mind, we decide to estimate the liquidity barrier b for each quarter using the 25th percentile as the cross-sectional distribution for the liquidity reserve without look-ahead bias. Put differently, we estimate the barrier based on historical information only and keep it constant throughout the future simulation. This approach assumes that firms ranging in the bottom quartile are exposed to a higher risk of financial distress. If the initial liquidity reserve is below the specified barrier *b*, we fund the firm liquid reserves with additional interest-bearing debt by the required amount (this adjustment is necessary for 45,006 firm quarters, however the average funding amount is 2.96 mio USD). Furthermore, given that the boundary condition is a critical parameter, we conduct a battery of additional robustness checks discussed in section 5.3, and all results remain valid. Figure 1 plots the estimated barrier *b* for the accounting based model over time. As shown, the barrier levels reveal a negative relation to periods with greater uncertainty and a positive trend over the sample period.

[Please insert Figure 1 about here]

The summary statistics for the individual parameters are presented in Table 3 separating solvent and performance-related distressed firms and the cross-sectional statistics. One observes that the sample firms are highly divergent with respect to size (parameter 1: quarterly sales ranging from less than one million dollars to less than four billion dollars), growth (parameter 2: quarterly sales growth ranging from less than -11% to more than 59%) and profitability (parameter 8: cost margins ranging from 36% to more than 350%). Moreover, the substantially lower sales median of approximately 38 million dollars compared with a mean of 266 million dollars demonstrates that there are few very large firms, while the majority are medium-sized firms. Finally, the initial *LR* position (parameter 16) for solvent and financially distressed firms differs significantly (the median for solvent (distressed) firms is 62 (9) million dollars), which also illustrates the diversity of firms in the sample. Most importantly, we find significant differences between solvent and distressed firms along the volatility of sales, sales growth rate and variable costs (parameters 6, 4, and 10). Figure 2 highlights the deterioration of the model parameters approaching the performance-related delisting.

[Please insert Table 3 about here]

[Please insert Figure 2about here]

5. EMPIRICAL ANALYSIS

In this section, we evaluate the performance of our distress prediction approach *S-Prob* along several dimensions and benchmark it against the prominent statistical models. Thus, section 5.1 provides a level playing field by employing accounting information for our approach and benchmark our model against other accounting based models, while section 5.2 is in the same spirit but allows our approach to take advantage of stock market data, and thus, it is put into perspective against well-established marked-based models.

5.1. Accounting-based Parameterization

5.1.1. Summary Statistics and Correlations

Table 4, Panel A presents the summary statistics for the key accounting-based models and for Standard & Poor's long-term credit rating, which is used to provide an alternative distress risk measure.

[Please insert Table 4 about here]

The results indicate that all models show the ability to separate performance-related delisted firms from non-delisting firms, as the average estimated distress probabilities for delisting firms are significantly higher. Looking at the medians, the *S-Prob* model reveals an even clearer ability to distinguish non-delisting firms (0.00) from delisting firms (0.45). Overall, the *Z-Prob* and *S-Prob* measures yield the highest estimated probabilities while the updated *Z-Prob*^{u}, *Z2-Prob*^{u} and *O-Prob*^{u} presents the lowest. As noted in Hillegeist et al. (2004), the results indicate that the re-estimated versions are not well calibrated and do not reflect changes in the underlying accounting ratios. For example, Franzen et al. (2007) report a negative trend in the relevant accounting ratios, which increases the probability of misclassification. Not surprisingly, all measures experience misclassifications with solvent (delisting) firms having high (low) probabilities in the 99th (1st) percentile. Table 4, Panel B displays the correlation measures for the different models. We focus on Spearman's correlation measure (below the diagonal), as the relationships are supposed to be monotonic but not necessarily linear, which is what Pearson's measure detects. While the original statistical *Z-Prob* and *O-Prob* models are strongly correlated (0.73), which is consistent with the value of 0.63 reported in Hillegeist et al. (2004), they are more modestly correlated with the *S-Prob* model (0.47 and 0.55). This finding shows that the model proposed in this study captures relevant aspects that are not included in the *Z-Prob* and *O-Prob* models. The lower correlations (0.20, 0.27 and 0.13) with the updated statistical models confirm this finding. These promising descriptive statistics are to be confirmed in the following analyses.

Before we turn to our main analysis, we perform a last sanity check by comparing our measure and the other established distress risk measures graphically with Standard & Poor's rating information.

[Please insert Figure 3 about here]

Thus, Figure 3 shows the median estimated distress probabilities for the different models according to the Standard & Poor's rating class. The predicted probabilities of all models increase with a deteriorated credit rating, as expected. Compared with the historic default rates from Standard & Poor's, the *S-Prob* model pictures the growing inherent risk of credit ratings with a steep increase from rating class "B" onwards to high probabilities levels in the substantial risk categories. However, while ratings provide a reasonably good measure for rough classification of firms according to their risk of failure, they are a poor measure of actual default probabilities as recently demonstrated by Hilscher and Wilson (2016). Furthermore, ratings are only available for a small subset of all firms. Consequently, we do not consider ratings for the further analyses.

[Please insert Figure 4 about here]

Turning to our main results, Figure 4 visually demonstrates the evolution of the default probabilities in the quarters that precede a forced delisting. One can clearly see the early warning ability of the *S-Prob* model. Compared with the two statistical models, the *S-Prob* model's estimated default probabilities start to increase substantially earlier, i.e., around five years before an actual delisting event compared to the three years for the *Z-Prob* and two years for the *O-Prob* measure. These early signals are economically highly relevant to avoid costly financial distress (Altman 1984; Warner 1977). One reason for the better performance of the *S-Prob* model for longer horizons might be the incorporation of the accounting volatility measures, since, e.g., Agarwal and Taffler (2008) also demonstrate that market-based models, which take volatility measures into account, perform better for longer horizons than statistical models.

While this section already demonstrates the benefits of *S-Prob* approach, the absolute probability levels are less relevant for the following cross-sectional discrimination tests, which therefore offer another dimension of model comparison.

5.1.2. Accuracy

When evaluating credit risk models, two types of errors influence a model's quality. First, bankrupt firms can be classified as non-bankrupt, i.e., a type I error (false negative). Second, non-bankrupt firms might be classified as bankrupt, thereby committing a type II error (false positive). Type I errors are associated with higher costs, which is why they are commonly considered as less desirable (Agarwal and Taffler 2008; Altman et al. 1977), while type II errors cause hypothetical loss of profits, interests and fees.[15] The methodology for the original *Z-Score* and *O-Score* models focus on a specific cutoff by minimizing the type I and/or type II error. In contrast, all estimates in this study are viewed as continuous measures (Hillegeist et al. 2004; Reisz and Perlich 2007). This is desirable because corporate distress is not a well-defined dichotomy in reality. Loan officers, for example, make continuous decisions at what rate to lend (Hillegeist et al. 2004). Hence, instead of directly counting the number of misclassified firms for a specific cutoff point, we evaluate the discriminatory capacity using a prevalent analytic accuracy measure, the area under the receiver operating curve (AUROC) (Agarwal and Taffler 2008; Caskey et al. 2012; Chava and Jarrow 2004; Reisz and Perlich 2007; Vassalou and Xing 2004). The ROC curve is constructed by varying the cutoff points and plotting the true positive rates (correct classification of a financially distressed firm) versus the false positive rates (false classification of a solvent firm). To compute the AUROC measure, we follow the nonparametric approach by Hanley and McNeil (1982) and DeLong et al. (1988) based on sorting the firms by their probability of default estimates (from high to low) and assessing the number of actual defaults in the highest k-percentiles (k=1,…,100). The AUROC measure, which ranges from 0.0 (no discrimination), 0.5 (random model) to a maximum of 1.0 (perfect discrimination model), answers the question of how accurate the model is in predicting actual defaults and determining cross-sectional distress risk. The higher the curve rises towards the top left corner point, the higher is the area under the curve (AUROC) and the better is the discrimination power.

[15] For a discussion of the relative importance of a type I or type II error, see, for instance, Altman et al. (1977), Altman (1984), Andrade and Kaplan (1998) and Blöchlinger and Leippold (2006).

To obtain fundamental assessments of the model performances, Table 5 compares the AUROC results inferred from predicting actual performance-related delistings over a 5-year horizon (i) by longitudinal analysis over years (Table 5, Panel A), (ii) by cross-sectional and bootstrapped analysis over the sample period 1980-2010 (Table 5, Panel B and C).

[Please insert Table 5 about here]

Overall, the *S-Prob* model experiences the highest accuracy (AUROC=0.8271) followed by the updated *O-Prob^u* model (AUROC=0.8087), the original *O-Prob* model (AUROC=0.7828), the original *Z-Prob* (AUROC=0.7338), updated *Z-Prob^u* (AUROC=0.7144) and updated *Z2-Prob^u* (AUROC=0.6922). The most striking result is the declining accuracy of the statistical models over the period 1990-2010. While the *O-Prob^u* and *O-Prob* model performed remarkably well up to the beginning of 1990s, the accuracy of the statistical models subsequently declines. This finding might be due to varying number of bankruptcies and delistings over time, where a higher number corresponds to a lower accuracy ratio (see Table 1). The findings also imply deterioration in the predictive power of accounting variables indicating unstable coefficients and miscalibration, which is consistent with the findings of Beaver et al. (2005), Hillegeist et al. (2004) and Begley et al. (1996) and explains the lower cross-sectional discrimination power of the re-estimated models.

Although Table 5, Panel B documents that the *O-Prob^u* model clearly outperforms the *Z-Prob, Z2-Prob* and even the *Z-Prob^u* specifications in terms of accuracy power, updating the coefficients is not a panacea as seen by the worse accuracy for the *Z-Prob^u* and *Z2-Prob^u* models. The results are consistent with information content analysis by Hillegeist et al. (2004). Broadly speaking, the accuracy of statistical models is widely dispersed, depending highly on the sample as well as the forecast horizon. In this sense, Agarwal and Taffler (2008) report an AUROC value of 0.89 (1-year forecast horizon) for the UK-based *Z-Score* model compared to the AUROC result of 0.7794 (1-year forecast horizon) and 0.6483 (5-year forecast horizon) reported by Reisz and Perlich (2007, Table 3). Chava and Jarrow (2004, Table 2) estimate a bootstrapped median AUROC of 0.8662 (1-year forecast horizon) based on yearly firm observations from 1991-1999 using a hazard model. Franzen et al. (2007, Table 7) report a mean cumulative accuracy ratio of 0.491 (1-year forecast horizon) for the R&D-adjusted *O-Prob* model using the sample period 1980-2003, which yields an equivalent AUROC of 0.7455. Similar, Jackson and Wood (2013) find an area under the ROC curve of 0.7801

(1-year forecast horizon) based on an annually sample for UK listed firms from 2000-2009. Overall, while this study uses a broader measure of distress (more than 3,000 delisting firms over the entire sample period compared to approximately 300 for most studies, as for example in Shumway 2001) and longer forecast horizons, the reported AUROC statistic is comparable to the prior literature.

In Table 5, Panel C the results are confirmed by employing 1,000 bootstrapped resamples to calculate the descriptive statistics for the AUROC and perform statistical inference (Chava and Jarrow 2004; Reisz and Perlich 2007). For the sake of completeness, we also sort by (i) cash, (ii) quick ratio, and (iii) assets-to-liability ratio, which are known to have predictive power but find low accuracies in (i) AUROC=0.2670, (ii) AUROC=0.3650, (iii) AUROC=0.3679, respectively.

[Please insert Figure 5 about here]

Figure 5 shows the AUROC of the models over time. While all accounting-based models increase their discrimination power in economic downturns (reflected by NBER recession periods), the *S-Prob* model is more accurate over time and outperforms more than 85 percent of our sample period. Interestingly, all models perform remarkably well during the financial crisis.

In the next section we investigate the incremental information content of the individual and combined distress probability estimates.

5.1.3. Test of Information Content

Following Campbell et al. (2008), Hillegeist et al. (2004) and Shumway (2001), we estimate a proportional (or dynamic) hazard model to evaluate the incremental explanatory power. We assume that the marginal probability of bankruptcy or failure over the next period follows a logistic distribution and is given by the following:

$$P_{i,t-1}\left(Y_{it}=1\right)=\frac{1}{1+\exp^{\left(-\alpha-\beta\cdot PD_{i,t-1}\right)}} \tag{17}$$

where Y is coded one if the firm delists in period t (and zero otherwise), and PD is the explanatory variable, which is the default probability estimate (coded as logit score) for the different models. The major criterion to compare the *S-Score* model to the benchmark models is the pseudo-R^2 as in

Shumway (2001). Additionally, we evaluate the Vuong test statistic for strictly non-nested model selection and the likelihood ratio statistic to compare the nested specifications. To examine the value of the models' default probabilities for longer forecast horizons, we vary them up to five years prior to delisting, as in Campbell et al. (2008).

Table 6, Panel A shows the results for the dynamic hazard model estimation to determine the information content of the models for a 1-year ahead prediction. Models 1-6 compare the individual explanation power, while models 7-12 focus on a combined explanatory power. Considered individually, all models have substantial explanatory power for actual delistings as the coefficients are significantly different from zero and strictly positive. Moreover, the pseudo-R^2s range from 0.0590 to 0.1293 individually. Overall, the magnitudes of the pseudo-R^2s are consistent with the findings in Xu and Zhang (2009) ranging from 0.07 to 0.16 or in Hillegeist et al. (2004) ranging from 0.07 to 0.12, where notably both studies include market-based prediction models. The *S-Score* and re-estimated *O-Score^u* models are more preferable using a 1-year forecast horizon (pseudo-R^2=0.1293 and pseudo-R^2=0.1200) and perform comparably well while all *Z-Score* variants are inferior in terms of the conveyed information (*Z-Score* pseudo-R^2=0.0608, *Z-Score^u* pseudo-R^2=0.0691 and *Z2-Score^u* pseudo-R^2=0.0590). In contrast to Hillegeist et al. (2004), the re-estimated *Z-Score^u* performs slightly better than the original *Z-Score*. As Xu and Zhang (2009) suggest, Table 6 also sets out the combination of the six measures (i.e., original and re-estimated models) to compare the incremental relevance carried by the stochastically driven *S-Score* distress prediction model. For the combined models, the *Z-Score, Z-Score^u* and *Z2-Score^u* coefficient estimates are insignificant, when combined with the *O-Score* variants. Looking at Models 10 - 12 versus the combined versions including the *S-Score* as explanatory variable (i.e., Models 7 - 9), the *S-Score* seems to explain information not contained in the traditional measures as documented by the higher pseudo-R^2.

Table 6, Panel B and C extend the forecast horizon of the dynamic hazard model to three and five years, respectively. As one expects, the pseudo-R^2 value decreases with longer forecast horizons. However, the results support the findings of the 1-year prediction. For a horizon of three years, for example, the *S-Score* model displays a pseudo-R^2 of 0.0736 compared to 0.0698 (0.0503) for the *O-Score^u* (*O-Score*) model. This finding is in line with the findings above that the *S-Score* model is superior in early prediction of defaults, which might be due to the inclusion of accounting based volatility measures. In all panels tabulated in Table 6, the combined models (model 7-9) offer the highest explanatory power. Surprisingly, the re-estimated *O-Score*^{u} loses part of its explanatory power but stays statistically significant (z=1.85) in the combined model versions referring to a 5-year forecast horizon. The high explanatory power of the *S-Score* for long-term financial distress predictions is confirmed by the pairwise Vuong-statistic offered in Panel D. Overall, the results suggest that *S-Score* model is preferable compared to the original and re-estimated versions of the *O-Score* and *Z-Score* variants in case of long-term forecast horizons.

[Please insert Table 6 about here]

5.2 Market-based Parameterization

In this section, we evaluate the performance of the *S-Prob*^{m} parameterization laid out in the earlier section 3.3. The question arises as to whether the accuracy and information content results hold in case we use an alternative market-based parameterization for the stochastically driven model and benchmark it against widespread used market-based financial distress prediction models (Bharath and Shumway 2008; Campbell et al. 2008; Correia et al. 2012; Crosbie and Bohn 2003).

First, we repeat the descriptive statistics, receiver operating characteristics and information content tests from section 5.1.1. to 5.1.3. As discussed above, we expect that market-information allows for better discrimination and additional explanatory power.

The summary statistics, accuracy analysis and information content tests are tabulated in the corresponding Tables 7, 8 and 9. The additional data requirements to calculate the alternative market-based distress measures, (i) *EDF, (ii) C-Prob,* (iii) *C-Prob*^{u}, (iv) *BhSh*^{u}, (v) *BhSh-DD*^{u}, and (vi) *Beaver*^{u}, reduce the sample size relative to the accounting-based sample from 330,274 (10,747) to 242,011 (9,438) firm quarter observations (firms) with observable probability estimations and ensure comparability between the default probability measures (see also Correia et al. 2012).

5.2.1. Summary Statistics and Accuracy (Market and Accounting Information)

The summary statistics for the market-based probabilities are presented in Table 7.

[Please insert Table 7 about here]

The mean and median values of all estimates show as expected higher estimates for financially distressed firms. Please recall that the models are based on different calibrations, and thus, the absolute amount is not comparable between models. The presentation mimics the accounting-based results in section 4 (please see also Hillegeist et al. 2004, p. 16 for a similar presentation). Among the correlations in Panel B, we recognize high rank correlations between the *C-Prob*^{u}, *Bhsh*^{u}, and *Beaver*^{u} probabilities (Spearman rank of 0.93 and higher), suggesting that these three models capturing similar information. The highest rank correlation between the *S-Prob*^{m} and alternative probability estimates is found for the *Beaver*^{u} (Spearman rank of 0.81); the lowest rank correlation is between the *S-Prob*^{m} and the *BhSh-DD*^{u} (Spearman rank of 0.21).

The most surprising fact revealed in the cross-sectional AUROC analysis (see Table 8, Panel A) is the low accuracy of the contingent claims models (*EDF* AUROC=0.7629, *BhSh-DD*^{u} AUROC=0.6609), which has multiple reasons. First, we recognize a low prediction accuracy during the 1980s and 1990s, increasing stepwise after the dot-com bubble 2000/2001. This finding is in line with the findings of Beaver et al. (2005) indicating a decline in the value-relevance of accounting ratios, while market information becomes more important in explaining distress risk. Second, the contingent claims framework implicitly assumes that liabilities mature in one year and neglect the liquidity risk (Hillegeist et al. 2004). Third, the contingent-claims model does not absorb possible inefficient market-information, particularly within a sample period including market downturns (Bharath and Shumway 2008; Das et al. 2009). However, the pure market-based models are clearly inferior to the mixed-information models in terms of a long-term prediction horizon.

[Please insert Table 8 about here]

In contrast, the *C-Prob*^{u} (AUROC=0.8419), *BhSh*^{u} (AUROC=0.8351) and *Beaver*^{u} (AUROC=0.8472) models perform remarkably well over the complete sample period and clearly outperform the *EDF* measure, while the *S-Prob*^{m} model (AUROC=0.8488) yields the highest discrimination accuracy 5 years prior to a performance-related delisting event. Prior findings conducted by Bauer and Agarwal (2014) imply an analogous ranking. Finally, the differences in the receiver operating characteristic of the original *C-Prob* (AUROC=0.8254) and the re-estimated *C-Prob*^{u} (AUROC=0.8419) support our premise that statistical models are not stable over time.[16]

5.2.2. Test of Information Content (Market and Accounting Information)

Table 9 contains the information content analysis of estimating a proportional hazard model with reference to our market-based sample and benchmarks. The tabulated models compare the explanatory power individually (models 1-7) and combined with the *S-Score*^{m} covariate (models 8-13). First, all covariates are significant in the univariate models 1-7 (1 year, 3 years and 5 years).

Consistent with the receiver operating characteristics in Table 8, the coefficient on the *EDF* model and the naïve version of the contingent claims model (*BhSh-DD*^{u}) lose most of their statistical significance as the prediction horizon is increased (3-year and 5-year).

For a one-year prediction horizon, the combined covariates are highly statistically significant. Interestingly, when the *S-Score*^{m} covariates are added to alternative specification for longer prediction horizons of three and five years, each of the covariates other than the *S-Score*^{m} loses significance. For the five-year prediction horizon, the alternative covariates are no longer statistically significant. In addition, the Vuong statistic in Panel D confirms that the *S-Score*^{m} measure become more important for long-term prediction horizons. We must acknowledge that the Vuong-test prefers the *C-Score*^{u} and *Beaver*^{u} one year prior to a subsequent delisting, mainly because extreme changes in the market-performance are incorporated with some delay into our underlying accounting- and market-based volatilities. However, recall that the *S-Score*^{m} is able to capture significantly incremental information content not captured by state-of-the-art financial distress prediction models even at the one-year prediction horizon.

[Please insert Table 9 about here]

5.3 Additional analyses

Our evidence so far supports the idea that the presented model captures various signals to predict the financial distress risk of a company, and it seems to be a step forward to provide a theoretical

[16] Supplementary analyses using the original coefficients of the *BhSh*^{u}*, BhSh-DD*^{u} *and Beaver*^{u} reveal a similar noticeable decline in the prediction accuracy.

framework for distress prediction. In this section, we attempt to further assess the performance of the stochastically driven model and ask whether the results also hold for alternative specifications.

(i) Alternative Prediction Horizon and Specifications

First, with respect to the long-term prediction horizon of this study, we extend the forecast horizon of the re-estimation procedures for the accounting-based and market-based benchmark models to calibrate the coefficients of the proportional hazard models 5 years prior to a performance-related delisting.[17] Balcaen and Ooghe (2006) note that the re-estimation horizon on observations should meet the horizon of the predictive classification statements (financially distressed/solvent) with reference to the parametric nature of the statistical framework. For the sake of brevity, we have not presented these comprehensive results here. For example, modifying the prediction horizon to 60 months ahead to calibrate coefficients for the *Beaver*^{u} model and investigating a 5-year ahead financial distress risk horizon reduces the accuracy in terms of the AUROC to 0.8170 (compared to 0.8472 for the one month-ahead model), consistent with the prior studies (Campbell et al. 2008; Tian et al. 2015). In addition, we test several alternative specifications for the original *Z-* and *O-Prob*, as well as the original *BhSh*^{u} and *Beaver*^{u} coefficients reported in the previous literature, but do not find significant improvements in the ability to correctly classifying financially distressed firms compared to our updated model versions (see for example: Begley et al. 1996; Hillegeist et al. 2004). Thus, regarding our primary focus to introduce and test a theoretical framework for financial distress risk, the results are promising compared in each dimension to the state-of-the-art prediction models.

(ii) Alternative Distress Barrier

With reference to the asset/debt implied boundary condition, we investigate a battery of unreported robustness tests. Specifically, we employ historical, industry-specific and firm-specific boundary conditions and test Parisian option barriers (Reisz and Perlich 2007) to indicate when the firm's financial condition enters a downward spiral. In addition, we calibrate a cross-sectional financial distress boundary using the iterative procedure introduced by Davydenko (2012b). Technically, the

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[17] Traditionally, most studies define a short-term prediction horizon (i.e., less than 16 months into the future) to calibrate their distress/default prediction models. For example the original *Z-Score* model by Altman (1968) was developed defining bankruptcy firms having an average lead time of the financial statements of 7½ months. Ohlson (1980) report an average lead time between the date of the fiscal year of the last relevant report and bankruptcy of 13 months. According to Crosbie and Bohn (2003), the time horizon specification for the contingent-claims model is regularly set to T=1 year even if the outcomes are used in predict distress/default for different forecast horizons.

boundary level is inferred by equalizing the type I error (false negative) and type II error (false positive) using a rolling or growing window approach to avoid look-ahead bias. To be precise, for each quarter, the type I error and type II error (in percentage) are calculated conditional on the presumed boundary condition. The barrier level that equalizes both error rates implies a minimum classification error. We also conduct a practical asset-implied barrier that equals 30% of the firm's average liquidity reserve 2 years prior to a performance-related delisting event but do not find significant improvements in the discriminatory power. However, since this criterion is somewhat arbitrary, we do not enhance our model in this regard. Finally, we employ the upper and lower 95% confidence bound for the 25th percentile of the liquidity reserve *LR* (see also Figure 1) to account for uncertainties in the underlying distribution. For example, the results do not change the inference for the *S-Prob*^{m} - accuracy (lower boundary AUROC=0.8489 (+0.0001), upper boundary AUROC=0.8486 (-0.0002)).

(iii) Industry-specific Accuracy Results

Table 10 reports the receiver operating characteristics (AUROC) for a five-year ahead forecast horizon according to Fama-French (2015) 10-industry classification for the accounting-based *S-Prob* model.[18] By its construction, the *S-Prob* covers significant industry effects and can improve the performance in 8 of 10 SIC-code classifications, which emphasize the importance of industry-specific effects (Chava and Jarrow 2004).

[Please insert Table 10 about here]

(iv) Delisting-codes specific Accuracy Results

With reference to the broad financial distress measure provided by CRSP delisting codes, Table 11 shows the predictive power for alternative ranges of delisting codes. To improve the indicator for firms that are forced to delist from the US stock markets for performance-related issues, we additionally classify firms delisted for reasons not available (CRSP delisting codes = 500) as financially distressed. Shumway (1997) notes that after 1987, CRSP assigned the three-digit delisting code "500" to the delisting category prior coded "5" that also summarized negative delisting causes and is commonly defined as performance-related delisting reason in the prior literature (Piotroski

[18] In a supplementary (untabulated) analysis, we also calculated the receiver operating characteristics (AUROC) according to Fama-French (2015) 48-industry classification for all models.

2000; Shumway 2001; Caskey et al. 2012). Supplementary analyses confirm that this category is associated with negative accounting and market-related figures (e.g., high leverage), and thus, it is reasonable to assume that this category also captures financially distressed firms. Finally, we exclude the delisting category "Delisted by current exchange - company request, deregistration (gone private)" (CRSP delisting codes = 573). This category thus serves as a placebo test. First, as expected, the accuracy and information content analysis document a worse performance of all financial distress prediction models in this category, which suggests that it does not reflect negative delisting issues. Second, supplementary summary statistics (not reported) for this delisting category document that the average firm that is assigned a delisting code 573 has positive excess returns, a positive quarterly net income and operating cash flows. Only 4 of 43 firms show continuing losses prior to their delistings.

While including the mixed-category "500" does not significantly affect the overall accuracy, the exclusion of firms assigned to the delisting category "573" improves the accuracy results of nearly all benchmark models. However, to ensure comparability, we operationalize the performance-related delisting range consistent to prior studies, i.e., (CRSP delisting codes = 400, 550-585).

[Please insert Table 11 about here]

6. CONCLUSION AND DISCUSSION

Understanding and projecting the financial health of companies is a challenging and important task. In this study, we present a theoretical framework that is also often employed in the equity valuation literature using stochastic processes to evaluate future firm development. Our approach offers several advantages compared to other, very often purely statistical techniques that have been employed to date. First, our approach is theoretically well grounded and can address the problems of the backward-looking perspective of accounting-based models. Second, our approach provides transparency regarding the exact distress mechanism, which is only implicitly captured by standard statistical techniques. Third, our approach flexibly links the bankruptcy literature to the features established in the equity valuation literature. Finally, our approach explicitly incorporates standard deviations of accounting number, i.e., accounting-based volatility measures, which seem to be important drivers to provide early financial distress warnings.

In addition to the theoretical advantages, our empirical implementation shows promising results. First, our generated distress probabilities fit the distribution of historic default rates reasonably well and thus provide early warning signals. Second, our measure is more accurate in discriminating performance-related delistings from solvent listings than the prominent state-of-the-art prediction models. Put differently, our measure outperforms other statistical procedures for longer time horizons, e.g., 3 and 5 years in advance, potentially allowing for early corrective action. Considered jointly with the significant explanatory power documented by the information content analysis, this study provides a useful approach towards a financial distress theory complementing the powerful existing models for short forecast horizons.

As a common critique to structural models, one possible drawback of our model is the strong underlying assumptions (Xu and Zhang 2009). The assumptions about firms' cash generating operating processes might be too simplified. However, the model successfully addresses several drawbacks of the statistical bankruptcy models and offers solid results. Obviously, the financial distress prediction based on stochastic processes has many degrees of freedom. It can therefore be regarded as a novel model class whose flexibility is a strength and an adequate response to the complex process of financial distress. Future research on this topic could help to model more detailed financing and investing policies of the firm and incorporate analysts' forecasts to refine the initializing parameterization.

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Table 1: Summary Delisting Statistics

Panel A: Performance-related Delisting Codes and Frequency (N= 330,274 firm quarters from 1980Q1 to 2010Q4)

CRSP delisting code	Delisting reasons	Unique firms	Firm quarters	%
400	Issue stopped trading as result of company liquidation.	2	20	0.05%
550	Delisted by current exchange - insufficient number of market makers.	156	1,454	3.50%
551	Delisted by current exchange - insufficient number of shareholders.	62	779	1.88%
552	Delisted by current exchange - price fell below acceptable level.	582	7,210	17.37%
560	Delisted by current exchange - insufficient capital, surplus, and/or equity	648	7,289	17.56%
561	Delisted by current exchange - insufficient (or non-compliance with rules of) float or assets.	345	3,964	9.55%
570	Delisted by current exchange - company request (no reason given).	188	2,424	5.84%
573	Delisted by current exchange - company request, deregistration (gone private).	43	648	1.56%
574	Delisted by current exchange - bankruptcy, declared insolvent.	379	4,952	11.93%
575	Delisted by current exchange - company request, offer rescinded, issue withdrawn by underwriter.	2	24	0.06%
580	Delisted by current exchange - delinquent in filing, non-payment of fees.	407	4,167	10.04%
581	Delisted by current exchange - failure to register under 12G of Securities Exchange Act.	37	428	1.03%
582	Delisted by current exchange - failure to meet exception or equity requirements.	96	1,114	2.68%
583	Delisted by current exchange - denied temporary exception requirement.	1	8	0.02%
584	Delisted by current exchange - does not meet exchange's financial guidelines for continued listing.	460	6,153	14.82%
585	Delisted by current exchange - protection of investors and the public interest.	76	872	2.10%
400,550-585	Delisting - all reasons	3,483	41,506	100.00%

This table lists the financial distress-related delisting reasons and their frequency and percentages of firm quarters in the sample. Generally, CRSP delisting codes 400-499 denote liquidations and 500-599 denote issues dropped from the stock exchange. For the purpose of this study, performance-related delisting codes 400 and 550-585 are considered as described in section 4.1. In total, there are 3,483 delisted firms (with 41,506 firm quarter observations) having distress-related information 20 quarters ahead of delisting. Note that after 1987, the CRSP assigned the three digit delisting code "500" to the delisting category prior coded "5" (Shumway 1997). In addition, the delisting category "572" is going to be discontinued and replaced with specific codes in the 400-range.

(continued on next page)

(Table 1 continued)

Panel B: Distribution of delisting's (N= 330,274 firm quarters from 1980Q1 to 2010Q4)

Year	# Traded firm's	# Delisted firm's	(%) Delisting rate	# Firm quarter observations	# distressed firm quarters	(%) Delisting firm quarter rate
1980	1,649	3	0.2%	5,799	127	2.2%
1981	1,602	9	0.6%	5,551	140	2.5%
1982	1,977	6	0.3%	5,902	213	3.6%
1983	2,799	12	0.4%	8,782	728	8.3%
1984	2,923	55	1.9%	10,052	1,064	10.6%
1985	3,042	80	2.6%	10,264	1,170	11.4%
1986	3,073	128	4.2%	10,574	1,360	12.9%
1987	3,029	90	3.0%	10,402	1,546	14.9%
1988	3,121	93	3.0%	10,568	1,709	16.2%
1989	3,170	118	3.7%	11,122	1,830	16.5%
1990	3,119	143	4.6%	10,997	1,679	15.3%
1991	3,115	167	5.4%	11,037	1,566	14.2%
1992	3,080	198	6.4%	10,946	1,260	11.5%
1993	3,216	88	2.7%	11,371	1,351	11.9%
1994	3,400	104	3.1%	11,991	1,607	13.4%
1995	3,652	96	2.6%	12,717	1,790	14.1%
1996	3,805	107	2.8%	13,389	2,200	16.4%
1997	3,952	125	3.2%	13,695	2,515	18.4%
1998	4,001	210	5.2%	13,797	2,687	19.5%
1999	3,811	213	5.6%	13,277	2,445	18.4%
2000	3,585	179	5.0%	12,642	2,287	18.1%
2001	3,434	254	7.4%	11,898	1,763	14.8%
2002	3,327	202	6.1%	11,882	1,347	11.3%
2003	3,128	168	5.4%	11,357	952	8.4%
2004	3,001	56	1.9%	11,086	1,046	9.4%
2005	2,910	86	3.0%	10,564	955	9.0%
2006	2,806	37	1.3%	10,281	921	9.0%
2007	2,757	27	1.0%	9,935	1,031	10.4%
2008	2,644	91	3.4%	9,670	923	9.5%
2009	2,564	107	4.2%	9,501	686	7.2%
2010	2,486	63	2.5%	9,225	608	6.6%

This table provides the distribution and summary statistics for the sample period (1980-2010). Traded firms comprise all unique observable firms in the sample, the number of delisted firms is the total number of firms delisted in a specific year and the number of firm quarters equals the total number of observations (traded and financially distressed firms). A firm quarter is considered as delisting firm quarter if the company is delisted in the next 20 quarters ahead. Note that 168 firms have been delisted after 2010 (e.g., 2011-2015). In total, the sample comprises 3,483 delisted firms with non-missing data (i.e., delistings with CRSP delisting codes 400, 550-585) representing 41,506 financially distressed firm quarters.

Table 2: Estimation of Parameters

No.	Label	Description	Measurement (abbreviations are COMPUSTAT mnemonics)
Sales dynamics			
1	S	= initial sales	= quarterly firm sales (saleq)
2	g_0	= initial sales growth rate	= estimated by the intercept (α) of a recursive least squares rolling firm-by-firm regression model (with 7 lagged firm quarters): $gs_{it} = \alpha + \beta gs_{it-1} + \varepsilon_{it}$; where $gs_{it} = \left(\frac{saleq_{it}}{saleq_{it-1}} - 1 \right)$
3	$\bar{g}$	= long-term sales growth rate	= 0.0
4	$\sigma_{g,0}$	= initial volatility of the sales growth rate	= estimated by the standard error of the intercept $(\hat{\sigma}_\alpha)$ of a recursive least squares rolling firm-by-firm regression model (with 7 lagged firm quarters): $gs_{it} = \alpha + \beta gs_{it-1} + \varepsilon_{it}$; where $gs_{it} = \left(\frac{saleq_{it}}{saleq_{it-1}} - 1 \right)$
5	$\overline{\sigma_g}$	= long-term volatility of sales growth rate	= 0.0
6	$\sigma_{S,0}$	= initial sales volatility	= estimated by the root mean squared error $(\hat{\sigma}_\varepsilon)$ of a recursive least squares rolling firm-by-firm regression model (with 7 lagged firm quarters): $gs_{it} = \alpha + \beta gs_{it-1} + \varepsilon_{it}$; where $gs_{it} = \left(\frac{saleq_{it}}{saleq_{it-1}} - 1 \right)$
7	$\bar{\sigma}_S$	= long-term volatility of sales	= median sales volatility (with 7 lagged firm quarters) [per quarter]
Cost dynamics			
8	c_0	= initial variable cost rate	= [saleq - (oiadpq + dpq + xrdq] / saleq
9	$\bar{c}$	= long-term variable cost	= median variable cost (with 7 lagged firm quarters) [per quarter]
10	$\sigma_{c,0}$	= initial volatility of variable costs	= estimated by the root mean squared error $(\hat{\sigma}_\varepsilon)$ of a recursive least squares rolling firm-by-firm regression model (with 7 lagged firm quarters): $vk_{it} = \alpha + \beta vk_{it-1} + \varepsilon_{it}$; where $vk_{it} = \frac{cogsq_{it} + xsgaq_{it}}{saleq_{it}}$
11	$\bar{\sigma}_c$	= long-term volatility of variable costs	= median volatility of variable costs (with 7 lagged firm quarters) [per quarter]

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(Table 2 continued)

Other parameters			
12	κ	= speed of adjustment	= estimated by the slope coefficient $(\beta/4)$ using an industry-specific (Fama-French (2015) 48-industry classification) least squares firm-by-industry regression model (re-estimated each year): $\ln\left(\frac{\frac{sale_{it}-sale_{it-1}}{sale_{it-1}}}{\frac{sale_{it-1}-sale_{it-2}}{sale_{it-2}}}\right)=\alpha+\beta\cdot\ln\left(\frac{sale_{it-1}-sale_{it-2}}{sale_{it-2}}\right)+\varepsilon_{it}$ $where\ t=1980,\ldots,2010$
13	*tax*	= tax rate	= top federal statutory corporate tax rate according to Nissim/Penman (2001, p. 151)
14	r_f	= risk-free rate	= 3-Month US-Treasury Bill Rate
15	r_{LTD}	= interest on debt	$= \frac{xint}{(dlc+dltt)}$
Balance sheet positions			
16	*LR*	= liquidity reserve	= Mean(Liquidity Risk (Illiquidity); Solvency Risk (Insolvency)): 0.5·(actq + unused credit line[a] – lctq)+ 0.5·(atq + unused credit line[a] –ltq)
17	*cash*	= interest-bearing cash and cash equivalents	= cheq
18	*LTD*	= long-term interest-bearing liabilities	= dlttq+(additional debt)

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(Table 2 continued)

COMPUSTAT					
Quarterly data (q)			*Annual data (a)*		
item number	mnemonic	description	item number	mnemonic	description
#1	xsgaq	Selling, General, and Administrative Expenses	#12	sale	Sales (Net)
#2	saleq	Sales (Net)	#15	xint	Interest and Related Expense – Total
#30	cogsq	Cost of Goods Sold	#34	dlc	Debt in Current Liabilities
#36	cheq	Cash and Equivalents	#41	cogs	Cost of Goods Sold
#37	rectq	Receivables – Total	#142	dltt	Long-Term Debt – Total
#38	invtq	Inventories – Total	#189	xsga	Selling, General, and Administrative Expenses
#39	acoq	Current Assets – Other			
#40	actq	Current Assets – Total (as sum of cheq, acoq, invtq and rectq)			
#45	dlcq	Debt in Current Liabilities			
#46	apq	Accounts Payable			
#49	lctq	Current Liabilities – Total			
#51	dlttq	Long-Term Debt – Total			
#44	atq	Assets – Total			
#54	ltq	Liabilities - Total			

This table presents the estimators for the different model parameters on a quarterly (yearly) basis (using COMPUSTAT mnemonics for reference).

[a]The unused credit line is estimated using the median ratio of lineun/atq times according to the Fama-French (2015) 10-industry classification and the indicator for unused credit lines provided by Sufi (2009).

Table 3: Initial Parameters

Univariate statistics (N=330,274 firm quarters from 1980Q1 to 2010Q4)								
No.		Status	N	Mean	Median	Std.dev.	1%	99%
	Sales dynamics							
1	initial sales	Solvent	288,768	296.192	47.016	918.977	0.379	4203.000
		Distressed	41,506	62.580	7.321	293.925	0.039	877.103
		Full sample	330,274	266.833	38.176	869.045	0.196	3939.701
2	initial sales growth rate	Solvent	288,768	0.066	0.041	0.108	-0.098	0.533
		Distressed	41,506	0.094	0.043	0.176	-0.189	0.793
		Full sample	330,274	0.070	0.041	0.119	-0.113	0.594
3	long-term sales growth rate	Solvent	288,768	0.000	0.000	0.000	0.000	0.000
		Distressed	41,506	0.000	0.000	0.000	0.000	0.000
		Full sample	330,274	0.000	0.000	0.000	0.000	0.000
4	initial volatility of the sales growth rate	Solvent	288,768	0.077	0.053	0.078	0.011	0.429
		Distressed	41,506	0.132	0.089	0.119	0.015	0.558
		Full sample	330,274	0.084	0.057	0.086	0.011	0.465
5	long-term volatility of sales growth rate	Solvent	288,768	0.000	0.000	0.000	0.000	0.000
		Distressed	41,506	0.000	0.000	0.000	0.000	0.000
		Full sample	330,274	0.000	0.000	0.000	0.000	0.000
6	initial sales volatility	Solvent	288,768	0.199	0.137	0.199	0.023	1.066
		Distressed	41,506	0.336	0.232	0.293	0.037	1.304
		Full sample	330,274	0.216	0.146	0.217	0.024	1.143
7	long-term volatility of sales	Solvent	288,768	0.163	0.166	0.014	0.131	0.184
		Distressed	41,506	0.167	0.170	0.011	0.137	0.184
		Full sample	330,274	0.163	0.167	0.014	0.131	0.184
	Cost dynamics							
8	initial variable cost rate	Solvent	288,768	0.926	0.883	0.814	0.351	2.416
		Distressed	41,506	1.407	0.978	2.113	0.477	11.353
		Full sample	330,274	0.986	0.893	1.080	0.360	3.610
9	long-term variable cost	Solvent	288,768	0.893	0.892	0.015	0.861	0.917
		Distressed	41,506	0.894	0.892	0.014	0.864	0.917
		Full sample	330,274	0.893	0.892	0.015	0.861	0.917
10	initial volatility of variable costs	Solvent	288,768	0.109	0.033	0.402	0.003	1.701
		Distressed	41,506	0.381	0.089	0.913	0.002	5.132
		Full sample	330,274	0.143	0.036	0.504	0.002	2.849
11	long-term volatility of variable costs	Solvent	288,768	0.041	0.041	0.006	0.022	0.055
		Distressed	41,506	0.042	0.042	0.005	0.025	0.055
		Full sample	330,274	0.041	0.041	0.006	0.022	0.055

(continued on next page)

(Table 3 continued)

	Other parameters							
12	speed of adjustment	Solvent	288,768	0.127	0.126	0.031	0.051	0.220
		Distressed	41,506	0.124	0.123	0.030	0.054	0.205
		Full sample	330,274	0.126	0.126	0.031	0.051	0.217
13	tax rate	Solvent	288,768	0.370	0.350	0.043	0.340	0.460
		Distressed	41,506	0.363	0.350	0.037	0.340	0.460
		Full sample	330,274	0.369	0.350	0.043	0.340	0.460
14	risk-free rate	Solvent	288,768	0.012	0.012	0.007	0.000	0.035
		Distressed	41,506	0.012	0.012	0.006	0.000	0.025
		Full sample	330,274	0.012	0.012	0.007	0.000	0.035
15	interest rate on debt	Solvent	288,768	0.022	0.020	0.020	0.000	0.110
		Distressed	41,506	0.028	0.023	0.022	0.000	0.110
		Full sample	330,274	0.023	0.020	0.020	0.000	0.110
	Balance sheet positions							
16	liquidity reserve	Solvent	288,768	329.671	62.484	911.680	2.593	5010.500
		Distressed	41,506	50.219	9.085	246.339	2.289	679.198
		Full sample	330,274	294.552	49.480	861.924	2.466	4636.000
17	interest-bearing cash and cash equivalents	Solvent	288,768	92.044	11.072	294.739	0.005	1718.000
		Distressed	41,506	17.756	1.337	106.176	0.000	284.266
		Full sample	330,274	82.709	8.615	279.244	0.002	1568.646
18	interest-bearing liabilities	Solvent	288,768	281.646	20.491	889.594	0.000	5355.100
		Distressed	41,506	101.064	8.252	443.808	0.000	1734.336
		Full sample	330,274	258.952	17.662	848.681	0.000	4968.513
a	unused credit line	Solvent	288,768	17.580	0.000	71.058	0.000	361.934
		Distressed	41,506	4.455	0.000	88.133	0.000	109.664
		Full sample	330,274	15.931	0.000	73.551	0.000	339.418

This table provides the summary statistics for the main *S-Score* model variables (see Table 2 for the calculation). All rates are quarterly growth rates and the balance sheet positions are in million dollars. There are 330,274 firm quarter observations over the period 1980Q1 to 2010Q4 (288,768 solvent firm quarters and 41,506 financially distressed firm quarters). Financially distressed firm quarters are indicated if the firm was delisted 20 quarter ahead. For the interest rate on debt 17,007 observations were set to zero as no debt expense was recorded. For the liquidity reserve, 45,006 values below the barrier *b* were funded by additional external debt to allow the stochastic processes to be initialized as described in section 4.2. All differences are statistically significant (at the 1%-level) based on a t-test (two-sided) of the means between the solvent and financial distressed sample (we confirm the results by testing the differences of the medians with the Wilcoxon rank-sum test yielding the same results).

Table 4: Descriptive Statistics

Panel A: Univariate summary statistics (solvent vs. financially distressed firm quarters from 1980Q1-2010Q4)

Variable	Status	Mean	Median	Std.dev.	1%	99%	N
Rating	Solvent	10.34	11.00	3.63	2.00	17.00	57,121
	Financially Distressed	14.38	14.00	2.30	8.00	21.00	3,843
Z-Prob	Solvent	0.16	0.10	0.20	0.00	0.99	288,768
	Financially Distressed	0.38	0.31	0.31	0.00	1.00	41,506
O-Prob	Solvent	0.05	0.01	0.11	0.00	0.63	288,768
	Financially Distressed	0.21	0.10	0.25	0.00	0.98	41,506
S-Prob	Solvent	0.10	0.00	0.23	0.00	1.00	288,768
	Financially Distressed	0.47	0.45	0.39	0.00	1.00	41,506
Z-Prob^u	Solvent	0.03	0.02	0.04	0.00	0.17	288,768
	Financially Distressed	0.07	0.03	0.13	0.00	0.74	41,506
O-Prob^u	Solvent	0.02	0.01	0.03	0.00	0.16	288,768
	Financially Distressed	0.07	0.04	0.08	0.00	0.41	41,506
Z2-Prob^u	Solvent	0.02	0.02	0.03	0.00	0.11	288,768
	Financially Distressed	0.05	0.03	0.08	0.00	0.48	41,506

Panel B: Correlations (Pearson above, Spearman rank below the diagonal)

Variable	Financial Distressed	Rating	Z-Prob	O-Prob	S-Prob	Z-Prob^u	O-Prob^u	Z2-Prob^u
Financially Distressed		0.27	0.30	0.27	0.31	0.20	0.29	0.17
Rating	0.28		0.58	0.39	0.36	0.28	0.47	0.23
Z-Prob	0.26	0.60		0.63	0.54	0.55	0.52	0.47
O-Prob	0.26	0.57	0.73		0.63	0.48	0.60	0.39
S-Prob	0.26	0.49	0.47	0.55		0.47	0.44	0.42
Z-Prob^u	0.17	0.31	0.63	0.45	0.20		0.47	0.97
O-Prob^u	0.23	0.68	0.53	0.68	0.37	0.55		0.42
Z2-Prob^u	0.14	0.23	0.52	0.36	0.13	0.98	0.50	

This table reports summary statistics for solvent vs. financially distressed observations by model. Panel A presents summary statistics of the evaluated model outcomes. RATING (=Standard & Poor's Credit Ratings, coded from 1 (“AAA”) to 21 (“D” or "SD") based on the COMPUSTAT item splticrm, Z-PROB (=original Altman Z-Score model 1968)), O-PROB (=original Ohlson (1980) O-Score model No. 1), S-PROB are the probability outcomes from the financial distressed prediction model using stochastic processes, Z-PROB^u (=updated Z-Score model by a logistic regression), O-PROB^u (=updated O-Score model probabilities by a logistic regression), Z2-PROB^u (=updated revised accounting-based Z-Score model by a logistic regression). The updated scores are transformed into probabilities by the standard logit transformation (1/(1+exp(-score)). The total number N of observations are 330,274 firm quarters / 10,747 firms over the sample period 1980Q1 to 2010Q4. FINANCIALLY DISTRESSED represents an indicator variable, which is one if delisting (indicated by CRSP delisting codes 400, 550-585) occurs within the next 20 quarters (i.e., a delisting firm quarter is determined if the firm experience a delisting within the next 20 quarters as defined in Table 1). The differences in probabilities of the solvent and financial distressed samples are significant at the 1%-level using a t-test and the Wilcoxon rank sum test (two-tailed). Note that for the Rating variable the sample size is reduced to 60,964 firm quarters / 2,243 firms covered by Standard & Poor’s credit ratings and CRSP delisting codes. Panel B shows the corresponding Pearson correlation above and the Spearman rank correlation below the diagonal for the evaluated models.

Table 5: Comparative Receiver Operating Characteristic (ROC) and Area Under the Curve (AUROC)

Panel A: Receiver Operating Characteristic (ROC): Area under the Curve (AUROC) by year (from 1980-2010)

	# firm quarters		S-Prob		Z-Prob		O-Prob		Z-Prob[u]		O-Prob[u]		Z2-Prob[u]	
Year	solvent	financially distressed	AUROC	SE	AUROC	SE	AUROC	SE	AUROC	SE	AUROC	SE	AUROC	SE
1980	5,672	127	0.8287	0.0207	0.8307	0.0182	0.8784	0.0154	0.7866	0.0212	0.8249	0.0193	0.7761	0.0221
1981	5,411	140	0.8087	0.0204	0.8389	0.0168	0.8717	0.0148	0.7577	0.0219	0.8326	0.0164	0.7566	0.0213
1982	5,689	213	0.8461	0.0146	0.7777	0.0196	0.8647	0.0139	0.7213	0.0205	0.8631	0.0121	0.7384	0.0193
1983	8,054	728	0.8660	0.0070	0.6898	0.0123	0.8240	0.0081	0.6961	0.0120	0.8360	0.0076	0.7255	0.0109
1984	8,988	1,064	0.8720	0.0054	0.7500	0.0092	0.8357	0.0063	0.7609	0.0084	0.8450	0.0060	0.7475	0.0084
1985	9,094	1,170	0.8532	0.0057	0.7650	0.0082	0.8286	0.0064	0.7562	0.0081	0.8496	0.0054	0.7381	0.0084
1986	9,214	1,360	0.8376	0.0054	0.7416	0.0079	0.8186	0.0060	0.7399	0.0075	0.8368	0.0052	0.7102	0.0079
1987	8,856	1,546	0.8365	0.0051	0.7316	0.0075	0.8033	0.0060	0.7331	0.0073	0.8432	0.0049	0.7027	0.0077
1988	8,859	1,709	0.8331	0.0050	0.7479	0.0071	0.7985	0.0060	0.7341	0.0071	0.8363	0.0050	0.7075	0.0074
1989	9,292	1,830	0.8464	0.0045	0.7571	0.0068	0.8021	0.0057	0.7455	0.0067	0.8450	0.0047	0.7152	0.0070
1990	9,318	1,679	0.8463	0.0047	0.7633	0.0070	0.8037	0.0059	0.7409	0.0072	0.8490	0.0047	0.7052	0.0076
1991	9,471	1,566	0.8466	0.0050	0.7472	0.0076	0.7911	0.0063	0.7349	0.0076	0.8452	0.0050	0.7040	0.0080
1992	9,686	1,260	0.8372	0.0057	0.7130	0.0088	0.7760	0.0072	0.7140	0.0086	0.8331	0.0057	0.6935	0.0088
1993	10,020	1,351	0.8334	0.0056	0.6947	0.0087	0.7616	0.0072	0.7267	0.0082	0.8281	0.0055	0.7121	0.0084
1994	10,384	1,607	0.8244	0.0055	0.7018	0.0077	0.7559	0.0066	0.7104	0.0075	0.8140	0.0054	0.6959	0.0076
1995	10,927	1,790	0.8024	0.0055	0.7045	0.0073	0.7479	0.0064	0.7153	0.0069	0.7868	0.0056	0.7041	0.0069
1996	11,189	2,200	0.8047	0.0050	0.6926	0.0067	0.7418	0.0060	0.7122	0.0064	0.7892	0.0051	0.7000	0.0065
1997	11,180	2,515	0.7973	0.0049	0.7436	0.0057	0.7597	0.0054	0.7287	0.0057	0.7859	0.0050	0.7083	0.0059
1998	11,110	2,687	0.8025	0.0045	0.7434	0.0055	0.7563	0.0052	0.7245	0.0056	0.7792	0.0049	0.7036	0.0058
1999	10,832	2,445	0.7951	0.0048	0.7153	0.0062	0.7583	0.0053	0.7245	0.0059	0.7803	0.0050	0.7067	0.0061
2000	10,355	2,287	0.8024	0.0049	0.7387	0.0062	0.7581	0.0055	0.7392	0.0060	0.7880	0.0051	0.7249	0.0062
2001	10,135	1,763	0.8314	0.0050	0.7759	0.0066	0.7867	0.0059	0.7574	0.0067	0.7988	0.0056	0.7393	0.0070
2002	10,535	1,347	0.8262	0.0059	0.7655	0.0073	0.7793	0.0068	0.7490	0.0078	0.7984	0.0062	0.7297	0.0081
2003	10,405	952	0.7972	0.0072	0.7282	0.0087	0.7481	0.0081	0.7145	0.0091	0.7715	0.0075	0.6970	0.0096
2004	10,040	1,046	0.8250	0.0063	0.7337	0.0086	0.7669	0.0075	0.7266	0.0088	0.8060	0.0065	0.7076	0.0091
2005	9,609	955	0.8303	0.0067	0.7513	0.0087	0.7734	0.0079	0.7425	0.0091	0.8139	0.0067	0.7151	0.0096
2006	9,360	921	0.8335	0.0068	0.7335	0.0095	0.7788	0.0081	0.7345	0.0095	0.8209	0.0066	0.7136	0.0098
2007	8,904	1,031	0.8576	0.0062	0.7595	0.0090	0.8073	0.0075	0.7790	0.0086	0.8509	0.0062	0.7616	0.0088
2008	8,747	923	0.8674	0.0062	0.7690	0.0087	0.7999	0.0078	0.7934	0.0084	0.8488	0.0065	0.7841	0.0086
2009	8,815	686	0.8639	0.0071	0.7593	0.0103	0.7722	0.0098	0.7706	0.0107	0.8299	0.0080	0.7549	0.0109
2010	8,617	608	0.8214	0.0092	0.7345	0.0112	0.7569	0.0106	0.7456	0.0120	0.8102	0.0095	0.7361	0.0121

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(Table 5 continued)

Panel B: Receiver Operating Characteristic (ROC): complete sample

Model:	# solvent	# financially distressed	AUROC	SE
S-Prob	288,768	41,506	0.8271	0.0011
Z-Prob	288,768	41,506	0.7338	0.0014
O-Prob	288,768	41,506	0.7828	0.0012
Z-Prob^{u}	288,768	41,506	0.7144	0.0014
O-Prob^{u}	288,768	41,506	0.8087	0.0011
Z2-Prob^{u}	288,768	41,506	0.6922	0.0015

Panel C: Receiver Operating Characteristic (ROC): bootstrapped samples

Model:	MEAN	MEDIAN	MIN	MAX	SE
S-Prob	0.8272	0.8271	0.8246	0.8297	0.0011
Z-Prob	0.7336	0.7336	0.7300	0.7370	0.0014
O-Prob	0.7826	0.7827	0.7797	0.7850	0.0012
Z-Prob^{u}	0.7143	0.7142	0.7113	0.7178	0.0014
O-Prob^{u}	0.8087	0.8087	0.8060	0.8117	0.0011
Z2-Prob^{u}	0.6922	0.6923	0.6879	0.6959	0.0015

Panel D: Proportion of financially distressed firm quarters outperformed by best model (N=41,506 delisted firm quarters and N=330,274 total firm quarters)

Model:	S-Prob	Z-Prob	O-Prob	Z-Prob^{u}	O-Prob^{u}	Z2-Prob^{u}
N	36,092	-	480	-	4,934	-
%	86.96%	0.00%	1.16%	0.00%	11.89%	0.00%

This table shows the Receiver Operating Characteristic curve for the evaluated models. Panel A reports the area under Receiver Operating Characteristic curve (AUROC) by years. The area under the ROC curve (AUROC) and SE are calculated following the nonparametric approach by DeLong et al. (1988), and Hanley and McNeil (1982). By definition, a firm quarter is considered as "financially distressed" if delisting occurs within the next 20 quarters. Panel B compares the AUROC for the entire sample (1980-2010). In Panel C we confirm the statistical inference and sample independence of the ROC curve by employing 1,000 bootstrap replications of the original sample to obtain bootstrap standard errors (SE). Panel D compares the number of firm quarters outperformed by the best model per year.

Table 6: Information content tests

Panel A: (1-year ahead prediction, firm quarters from 1980Q1-2010Q4, 327,166 solvent firm quarters vs. 3,108 financially distressed firm quarters)

Variables:	Model(1)	Model(2)	Model(3)	Model(4)	Model(5)	Model(6)	Model(7)	Model(8)	Model(9)	Model(10)	Model(11)	Model(12)
Constant	-4.132	-4.367	-3.246	-2.194	-1.378	-1.650	-3.739	-2.529	-2.621	-3.333	-1.319	-1.382
	(34.84)***	(36.74)***	(20.68)***	(7.58)***	(4.49)***	(4.25)***	(18.63)***	(6.03)***	(5.59)***	(19.22)***	(4.17)***	(3.50)***
coeff S-Score	0.255	.	.	.	.	.	0.191	0.158	0.159	.	.	.
	(11.01)***	.	.	.	.	.	(5.83)***	(4.55)***	(4.57)***	.	.	.
coeff Z-Score	.	0.273	.	.	.	.	0.073	.	.	0.074	.	.
	.	(8.13)***	.	.	.	.	(1.63)	.	.	(1.42)	.	.
coeff O-Score	.	.	0.516	.	.	.	0.142	.	.	0.447	.	.
	.	.	(10.05)***	.	.	.	(1.65)*	.	.	(6.25)***	.	.
coeff Z-Score[u]	.	.	.	0.718	.	.	.	-0.002	.	.	0.033	.
	.	.	.	(8.62)***	.	.	.	(0.02)	.	.	(0.25)	.
coeff O-Score[u]	.	.	.	.	0.864	.	.	0.472	0.502	.	0.856	0.889
	.	.	.	.	(9.88)***	.	.	(3.21)***	(3.47)***	.	(6.84)***	(7.44)***
coeff Z2-Score[u]	.	.	.	.	.	0.856	.	.	-0.058	.	.	-0.020
	.	.	.	.	.	(7.75)***	.	.	(0.39)	.	.	(0.13)
Pseudo-R^2	0.1293	0.0608	0.1026	0.0691	0.1200	0.0590	0.1424	0.1481	0.1483	0.1050	0.1255	0.1255
LR[a]	3,304	1,555	2,624	1,768	2,920	1,508	3,642	3,786	3,790	2,683	3,210	3,208

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(Table 6 continued)

Panel B: (3-year ahead prediction, firm quarters from 1980Q1-2010Q4, 327,166 solvent firm quarters vs. 3,108 financially distressed firm quarters)

Variables:	Model(1)	Model(2)	Model(3)	Model(4)	Model(5)	Model(6)	Model(7)	Model(8)	Model(9)	Model(10)	Model(11)	Model(12)
Constant	-4.194	-4.537	-3.585	-2.848	-2.057	-2.538	-3.902	-2.757	-2.808	-3.575	-2.040	-2.204
	(27.67)***	(27.20)***	(15.24)***	(6.32)***	(4.76)***	(4.34)***	(14.53)***	(4.93)***	(4.39)***	(14.85)***	(4.45)***	(3.96)***
coeff S-Score	0.207	.	.	.	.	.	0.170	0.129	0.130	.	.	.
	(7.07)***	.	.	.	.	.	(4.11)***	(2.98)***	(2.99)***	.	.	.
coeff Z-Score	.	0.183	.	.	.	.	0.022	.	.	-0.012	.	.
	.	(3.58)***	.	.	.	.	(0.37)	.	.	(0.18)	.	.
coeff O-Score	.	.	0.396	.	.	.	0.108	.	.	0.407	.	.
	.	.	(5.85)***	.	.	.	(0.94)	.	.	(4.47)***	.	.
coeff Z-Score^u	.	.	.	0.541	.	.	.	-0.016	.	.	-0.091	.
	.	.	.	(4.54)***	.	.	.	(0.10)	.	.	(0.56)	.
coeff O-Score^u	.	.	.	.	0.678	.	.	0.413	0.418	.	0.768	0.782
	.	.	.	.	(6.24)***	.	.	(2.59)***	(2.67)***	.	(5.27)***	(5.56)***
coeff Z2-Score^u	.	.	.	.	.	0.622	.	.	-0.035	.	.	-0.148
	.	.	.	.	.	(4.01)***	.	.	(0.19)	.	.	(0.75)
Pseudo-R^2	0.0736	0.0204	0.0503	0.0310	0.0698	0.0261	0.0772	0.0881	0.0881	0.0503	0.0782	0.0786
LR^a	1,287	357	879	543	1,112	457	1,350	1,448	1,449	880	1,367	1,374

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(Table 6 continued)

Panel C: (5-year ahead prediction, firm quarters from 1980Q1-2010Q4, 327,166 solvent firm quarters vs. 3,108 financially distressed firm quarters)

Variables:	Model(1)	Model(2)	Model(3)	Model(4)	Model(5)	Model(6)	Model(7)	Model(8)	Model(9)	Model(10)	Model(11)	Model(12)
Constant	-4.317	-4.739	-3.834	-3.362	-2.620	-3.101	-4.064	-3.228	-3.252	-3.794	-2.545	-2.679
	(22.69)***	(21.52)***	(12.50)***	(5.67)***	(4.69)***	(4.20)***	(12.19)***	(4.78)***	(4.25)***	(12.32)***	(4.38)***	(3.88)***
coeff S-Score	0.187	.	.	.	.	.	0.156	0.127	0.126	.	.	.
	(5.20)***	.	.	.	.	.	(3.09)***	(2.43)**	(2.43)**	.	.	.
coeff Z-Score	.	0.131	.	.	.	.	-0.013	.	.	-0.056	.	.
	.	(2.02)**	.	.	.	.	(0.17)	.	.	(0.70)	.	.
coeff O-Score	.	.	0.342	.	.	.	0.112	.	.	0.393	.	.
	.	.	(4.08)***	.	.	.	(0.79)	.	.	(3.60)***	.	.
coeff Z-Score^u	.	.	.	0.424	.	.	.	-0.057	.	.	-0.128	.
	.	.	.	(2.83)***	.	.	.	(0.33)	.	.	(0.67)	.
coeff O-Score^u	.	.	.	.	0.548	.	.	0.347	0.340	.	0.685	0.681
	.	.	.	.	(4.20)***	.	.	(1.85)*	(1.84)*	.	(4.05)***	(4.15)***
coeff Z2-Score^u	.	.	.	.	.	0.493	.	.	-0.056	.	.	-0.158
	.	.	.	.	.	(2.62)***	.	.	(0.26)	.	.	(0.68)
Pseudo-R^2	0.0550	0.0097	0.0346	0.0183	0.0455	0.0163	0.0569	0.0653	0.0652	0.0357	0.0568	0.0569
LR^a	685	121	431	228	490	203	708	768	767	444	707	708

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(Table 6 *continued)*

Panel D: Vuong-test statistics

S-Score	*Z-Score*	*O-Score*	*Z-Score^u*	*O-Score^u*	*Z2-Score^u*
Model(1) vs.	Model(2)	Model(3)	Model(4)	Model(5)	Model(6)
1-year ahead	130.29***	67.05***	117.90***	14.67***	125.58***
3-year ahead	112.48***	59.15***	84.43***	9.38***	92.96***
5-year ahead	104.29***	53.09***	77.51***	23.26***	81.29***

*/**/*** asterisks refer to significance at a 10%/ 5%/ 1% level for a two-sided test. Figures in brackets are the z-statistics.

This table shows the coefficients, z-statistics (in parentheses, which accounts for firm dependence between firm quarter observations), McFadden's (1974)-Pseudo-R^2 and the likelihood ratio statistic $LR=2(L_1-L_0)$, where L_1 is the maximized log likelihood for the unrestricted model, and L_0 is the maximized log likelihood for the restricted model with a constant only using a dynamic hazard models (as in Chava and Jarrow 2004; Shumway 2001). Due to the panel structure of the data there are fewer independent observations than assumed by a standard logit regression model. The panels compare the contribution of the S-SCORE estimation with the results from univariate and combined regressions results for 1, 3 and 5-year ahead predictions. The probabilities are converted into scores according to the following: $score_i = ln[prob_i/(1-prob_i)]$.

Panel D reports the Vuong LR test statistic results for strictly non-nested models (model(1) vs. model(2-6)). A positive LR test statistic indicates that the S-SCORE model (i.e., model(1)) is preferable. If the LR test statistic is negative the compared model is favored.

[a] in unreported likelihood ratio tests, we show that the S-SCORE model provides incremental information beyond (updated) combined Z_SCORE/O-SCORE models. The difference between the models (comparing nested models Model(7) vs. Model(10), Model(8) vs. Model(11) and Model(9) vs. Model(12)) is significant at the 1%- level for all forecast horizon (1, 3 and 5-year ahead predictions).

Table 7: Descriptive Statistics (Market and Accounting Information)

Panel A: Univariate summary statistics (Solvent vs. financially distressed firm quarters from 1980Q1-2010Q4)

Variable	Status	Mean	Median	Std.dev.	1%	99%	N
S-Prob^{m}	Solvent	0.17	0.03	0.26	0.00	0.99	210,565
	Financially Distressed	0.60	0.68	0.33	0.00	1.00	31,446
EDF	Solvent	0.06	0.00	0.16	0.00	0.84	210,565
	Financially Distressed	0.23	0.06	0.30	0.00	0.99	31,446
C-Prob	Solvent	0.00	0.00	0.05	0.00	0.08	210,565
	Financially Distressed	0.05	0.00	0.16	0.00	0.91	31,446
C-Prob^{u}	Solvent	0.00	0.00	0.01	0.00	0.02	210,565
	Financially Distressed	0.01	0.00	0.03	0.00	0.15	31,446
BhSh^{u}	Solvent	0.00	0.00	0.00	0.00	0.01	210,565
	Financially Distressed	0.01	0.00	0.02	0.00	0.10	31,446
BhSh-DD^{u}	Solvent	0.03	0.02	0.05	0.00	0.29	210,565
	Financially Distressed	0.07	0.02	0.11	0.00	0.48	31,446
Beaver^{u}	Solvent	0.00	0.00	0.01	0.00	0.03	210,565
	Financially Distressed	0.02	0.00	0.04	0.00	0.18	31,446

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(Table 7 continued)

Panel B: Correlations (Pearson above, Spearman rank below the diagonal)

Variable	Financially Distressed	S-Prob^{m}	EDF	C-Prob	C-Prob^{u}	BhSh^{u}	BhSh-DD^{u}	Beaver^{u}
Financially Distressed		0.47	0.31	0.20	0.29	0.25	0.26	0.28
S-Prob^{m}	0.41		0.49	0.28	0.39	0.33	0.33	0.39
EDF	0.31	0.63		0.40	0.43	0.39	0.66	0.39
C-Prob	0.38	0.73	0.78		0.78	0.65	0.35	0.51
C-Prob^{u}	0.40	0.80	0.68	0.75		0.73	0.36	0.66
BhSh^{u}	0.39	0.77	0.70	0.71	0.94		0.42	0.68
BhSh-DD^{u}	0.19	0.21	0.33	0.30	0.36	0.33		0.38
Beaver^{u}	0.40	0.81	0.71	0.76	0.95	0.93	0.33	

This table reports statistics for solvent vs. financially distressed observations by model. Panel A presents summary statistics of the evaluated market-based model outcomes compared to the S-PROB^{m} model including market information. EDF (Expected Default Frequency) is the default probability estimated monthly following the sequential-iterations algorithm specified by Crosbie and Bohn (2003) and Bharath and Shumway (2008).
C-PROB is defined as the monthly updated default probability using the original coefficient from the mixed-model No. 2 by Campbell et al. (2008). C-PROB^{u} is the corresponding model No. 2 (re-estimated by a growing window proportional hazard regression model, starting with month 1976|01).
BHSH^{u} / BHSH-DD^{u} are defined as in Bharath and Shumway (2008), model 7 / model 2 using the naïve alternative distance-to-default measure (re-estimated by a growing window proportional hazard regression model, starting with month 1976|01). BEAVER^{u} is the corresponding Beaver et al. (2012) combined model as defined in Correia et al. (2012) using accounting and market information (re-estimated by a growing window proportional hazard regression model, starting with month 1976|01). We winsorize all measures at the 1st and 99th percentile before re-estimating the logits. The area under the ROC Curve (AUROC) and its standard deviation (SE) is calculated following the nonparametric approach by DeLong et al. (1988), Hanley and McNeil (1982). By definition, a firm is considered as "delisted" if delisting occurs within the next 20 quarters ahead.
The total number N of observations are 242,011 firm quarters / 9,438 firms over the sample period 1980Q1 to 2010Q4 with non-missing values for the (market) default probabilities. FINANCIALLY DISTRESSED represents an indicator variable, which is one if delisting (indicated by CRSP delisting codes 400, 550-585) occurs within the next 20 quarters (i.e., a delisting firm quarter is determined if the firm experience a performance-related delisting within the next 20 quarters as defined in Table 1). The differences in probabilities of the solvent and financially distressed means are significant at the 1%-level using a t-test or the Wilcoxon rank sum test (two-tailed). Panel B shows the Pearson correlation above and the Spearman rank correlation below the diagonal for the evaluated models.

Table 8: Receiver Operating Characteristic (ROC) (Market and Accounting Information)

Panel A: Receiver Operating Characteristic (ROC): complete sample

Model:	# solvent	# delisted	AUROC	SE
S-Prob^m	210,565	31,446	0.8488	0.0011
EDF	210,565	31,446	0.7704	0.0014
C-Prob	210,565	31,446	0.8254	0.0012
C-Prob^u	210,565	31,446	0.8419	0.0011
BhSh^u	210,565	31,446	0.8351	0.0012
BhSh-DD^u	210,565	31,446	0.6609	0.0018
Beaver^u	210,565	31,446	0.8472	0.0011

Panel B: Receiver Operating Characteristic (ROC): bootstrapped samples

Model:	MEAN	MEDIAN	MIN	MAX	SE
S-Prob^m	0.8489	0.8488	0.8470	0.8523	0.0010
EDF	0.7708	0.7706	0.7677	0.7740	0.0013
C-Prob	0.8255	0.8255	0.8227	0.8285	0.0012
C-Prob^u	0.8419	0.8419	0.8388	0.8444	0.0011
BhSh^u	0.8350	0.8350	0.8323	0.8379	0.0012
BhSh-DD^u	0.6609	0.6609	0.6569	0.6651	0.0018
Beaver^u	0.8472	0.8472	0.8450	0.8492	0.0011

Panel A reports the area under Receiver Operating Characteristic curve (AUROC) for the S-PROB^m model (parameterized with additional market-information) compared to (mixed)-market models. The EDF (Expected Default Frequency) equals the default probability estimated monthly following the sequential-iterations algorithm of Crosbie and Bohn (2003) and Bharath and Shumway (2008). C-PROB is defined as the monthly updated default probability using the original coefficient vector from the mixed-model No. 2 by Campbell et al. (2008). C-PROB^u is the corresponding model No. 2 (re-estimated by a growing window proportional hazard regression model, starting with month 1976|01). BHSH^u / BHSH-DD^u are defined as in Bharath and Shumway (2008), model 7 / model 2 using the naïve alternative distance-to-default measure (re-estimated by a growing window proportional hazard regression model, starting with month 1976|01). BEAVER^u is the corresponding Beaver et al. (2012) combined model as defined in Correia et al. (2012) using accounting and market information (re-estimated by a growing window proportional hazard regression model, starting with month 1976|01). We winsorize all measures at the 1st and 99th percentile before re-estimating the logits. The area under the ROC Curve (AUROC) and its standard deviation (SE) is calculated following the nonparametric approach by DeLong et al. (1988), Hanley and McNeil (1982). By definition, a firm is considered as "delisted" if delisting occurs within the next 20 quarters ahead. In Panel B, we also confirm the statistical significance of the differences between the ROC curve by employing 1,000 bootstrap replications of the original sample to obtain bootstrap standard errors (SE) and performing t-tests.

Table 9: Information content tests (Market and Accounting Information)

Panel A: (1-year ahead prediction, firm quarters from 1980Q1-2010Q4, 239,475 solvent firm quarters vs. 2,536 financially distressed firm quarters)

Variables:	Model(1)	Model(2)	Model(3)	Model(4)	Model(5)	Model(6)	Model(7)	Model(8)	Model(9)	Model(10)	Model(11)	Model(12)	Model(13)
Constant	-4.398	-3.471	-1.689	0.289	0.654	-2.117	0.039	-3.818	-3.013	-1.375	-1.377	-2.923	-1.381
	(33.00)***	(22.50)***	(6.30)***	(0.72)	(1.50)	(6.66)***	(0.10)	(21.31)***	(8.09)***	(2.28)**	(2.12)**	(8.43)***	(2.49)**
coeff S-Score^m	0.381							0.328	0.299	0.219	0.239	0.354	0.203
	(10.81)***							(8.01)***	(6.87)***	(4.13)***	(4.71)***	(9.39)***	(3.79)***
coeff EDF-Score		0.266						0.139					
		(8.45)***						(3.92)***					
coeff C-Score			0.403						0.193				
			(10.79)***						(3.77)***				
coeff C-Score^u				0.743						0.473			
				(10.90)***						(4.86)***			
coeff BhSh-Score^u					0.733						0.431		
					(11.10)***						(4.54)***		
Coeff BhSh-DD-Score^u						0.723						0.413	
						(7.91)***						(4.27)***	
coeff Beaver-Score^u							0.701						0.471
							(11.20)***						(5.27)***
Pseudo R^2	0.1504	0.0973	0.1086	0.1618	0.1506	0.0633	0.1703	0.1717	0.1674	0.1821	0.1774	0.1722	0.1876
LR	2,742	1,775	1,980	2,949	2,746	1,154	3,105	3,131	3,052	3,320	3,234	3,139	3,421

(continued on the next page)

(Table 9 continued)

Panel B: (3-year ahead prediction, firm quarters from 1980Q1-2010Q4, 239,796 solvent firm quarters vs. 2,540 financially distressed firm quarters)

Variables:	Model(1)	Model(2)	Model(3)	Model(4)	Model(5)	Model(6)	Model(7)	Model(8)	Model(9)	Model(10)	Model(11)	Model(12)	Model(13)
Constant	-4.358	-3.744	-2.200	-0.594	-0.375	-2.843	-0.912	-3.982	-3.743	-2.196	-2.613	-3.520	-2.445
	(27.64)***	(16.06)***	(4.77)***	(1.04)	(0.59)	(5.11)***	(1.73)*	(15.78)***	(5.85)***	(2.56)**	(2.74)***	(6.25)***	(3.10)***
coeff S-Score^m	0.319							0.280	0.288	0.197	0.238	0.303	0.204
	(7.01)***							(5.25)***	(5.10)***	(2.75)***	(3.58)***	(6.35)***	(2.91)***
coeff EDF-Score		0.200						0.078					
		(5.14)***						(1.73)*					
coeff C-Score			0.331						0.082				
			(5.73)***						(0.98)				
coeff C-Score^u				0.592						0.327			
				(6.96)***						(2.48)**			
coeff BhSh-Score^u					0.577						0.241		
					(6.68)***						(1.82)*		
coeff BhSh-DD-Score^u						0.529						0.221	
						(3.66)***						(1.51)	
coeff Beaver-Score^u							0.539						0.286
							(7.00)***						(2.39)**
Pseudo R^2	0.0913	0.0498	0.0442	0.0901	0.0744	0.0220	0.0871	0.0974	0.0931	0.1039	0.0979	0.0956	0.1028
LR	1,074	586	521	1,060	876	259	1,025	1,146	1,096	1,223	1,152	1,125	1,210

(continued on the next page)

(Table 9 continued)

Panel C: (5-year ahead prediction, firm quarters from 1980Q1-2010Q4, 239,796 solvent firm quarters vs. 2,540 financially distressed firm quarters)

Variables:	Model(1)	Model(2)	Model(3)	Model(4)	Model(5)	Model(6)	Model(7)	Model(8)	Model(9)	Model(10)	Model(11)	Model(12)	Model(13)
Constant	-4.489	-4.140	-2.973	-1.343	-1.205	-3.558	-1.674	-4.351	-4.784	-2.948	-3.568	-4.141	-3.337
	(22.73)***	(12.83)***	(4.29)***	(1.81)*	(1.40)	(4.46)***	(2.39)***	(12.71)***	(4.89)***	(2.64)***	(2.80)***	(5.33)***	(3.19)***
coeff S-Score^m	0.280							0.265	0.294	0.190	0.237	0.273	0.211
	(4.91)***							(4.00)***	(4.12)***	(2.09)**	(2.82)***	(4.59)***	(2.39)**
coeff EDF-Score		0.149						0.027					
		(3.04)***						(0.48)					
coeff C-Score			0.250						-0.038				
			(3.00)***						(0.31)				
coeff C-Score^u				0.496						0.231			
				(4.73)***						(1.37)			
coeff BhSh-Score^u					0.480						0.126		
					(4.32)***						(0.72)		
coeff BhSh-DD-Score^u						0.377						0.090	
						(1.89)*						(0.46)	
coeff Beaver-Score^u							0.443						0.169
							(4.59)***						(1.10)
Pseudo R^2	0.0650	0.0251	0.0187	0.0588	0.0450	0.0094	0.0530	0.0657	0.0653	0.0706	0.0665	0.0656	0.0686
LR	514	199	148	465	356	74	419	520	517	559	527	519	543

(continued on the next page)

(Table 9 continued)

Panel D: Vuong-test statistics

S-Score^{m}	*EDF-Score*	*C-Score*	*C-Score*^{u}	*BhSh-Score*^{u}	*BhSh-DD-Score*^{u}	*Beaver-Score*^{u}
Model(1) vs.	Model(2)	Model(3)	Model(4)	Model(5)	Model(6)	Model(7)
1-year ahead	82.37***	137.50***	-20.38***	35.34***	140.01***	-28.19***
3-year ahead	80.53***	117.54***	12.14***	62.57***	115.41***	27.83***
5-year ahead	83.43***	104.06***	24.42***	63.55***	98.19***	43.98***

*/**/*** asterisks refer to significance at a 10%/ 5%/ 1% level for a two-sided test.

This table shows the coefficients, z-statistics (in parentheses, which accounts for firm dependence between firm quarter observations), McFadden's (1974)-Pseudo-R^2 and the likelihood ratio statistic LR=2(L_1-L_0), where L_1 is the maximized log likelihood for the unrestricted model, and L_0 is the maximized log likelihood for the restricted model with a constant only using a dynamic hazard models (Chava and Jarrow 2004; Shumway 2001). The sample is limited to non-missing observations of the default probabilities using market models (in total there are 242,336 firm quarter / 9,442 firm observations). Due to the panel structure of the data, there are fewer independent observations than assumed by a standard logit regression model. The panels compare the contribution of the S-SCORE^{m} estimation with the results from univariate and combined regressions results for 1 (Panel A), 3 (Panel B) and 5-year (Panel C) ahead predictions. The probabilities are converted into scores according to the following: $score_i = ln[prob_i/(1-prob_i)]$.

Panel D reports the Vuong LR test statistic results for strictly non-nested models (Model(1) vs. Model(2-7)). A positive LR test statistic indicates that the S-SCORE^{m} model (i.e., Model(1)) is preferable. If the LR test statistic is negative, the compared model is superior.

Table 10: Receiver Operating Characteristic by Industry

Receiver Operating Characteristic (ROC): Area under the Curve (AUROC) by Fama and French 10-Industry classification for a 5-year ahead prediction,

Model:				S-Prob		Z-Prob		O-Prob		Z-Prob^{u}		O-Prob^{u}		Z2-Prob^{u}	
FFI10	Desc	solvent	distressed	AUROC	SE	AUROC	SE	AUROC	SE	AUROC	SE	AUROC	SE	AUROC	SE
1	NoDur	24,210	3,382	0.8041	0.0041	0.7342	0.0051	0.7561	0.0047	0.6907	0.0048	0.8128	0.0037	0.6716	0.0050
2	Durbl	10,554	1,736	0.8308	0.0051	0.7423	0.0071	0.7839	0.0060	0.7253	0.0069	0.8181	0.0055	0.7124	0.0071
3	Manuf	60,637	6,440	0.8373	0.0026	0.7649	0.0036	0.8178	0.0029	0.7445	0.0034	0.8515	0.0024	0.7230	0.0035
4	Enrgy	16,218	2,539	0.8466	0.0042	0.7188	0.0065	0.7794	0.0050	0.6331	0.0070	0.7910	0.0048	0.6033	0.0070
5	HiTec	67,179	9,613	0.8488	0.0020	0.7430	0.0030	0.8112	0.0023	0.7450	0.0029	0.8208	0.0022	0.7183	0.0031
6	Telcm	6,978	1,277	0.7845	0.0066	0.6701	0.0091	0.6770	0.0083	0.7118	0.0085	0.7447	0.0069	0.6969	0.0087
7	Shops	41,135	6,977	0.8153	0.0027	0.7519	0.0033	0.7637	0.0032	0.6811	0.0036	0.7900	0.0028	0.6623	0.0036
8	Hlth	26,871	2,908	0.8350	0.0038	0.7335	0.0055	0.7939	0.0042	0.7470	0.0050	0.8135	0.0041	0.7162	0.0054
9	Utils	833	61	0.9426	0.0085	0.8566	0.0369	0.8040	0.0381	0.8163	0.0334	0.8595	0.0231	0.7552	0.0350
10	Other	32,833	5,454	0.7981	0.0032	0.7065	0.0041	0.7445	0.0037	0.6952	0.0042	0.7813	0.0034	0.6744	0.0043
n/a	n/a	1,320	1,119	0.7492	0.0098	0.6788	0.0113	0.6867	0.0107	0.7949	0.0090	0.8034	0.0087	0.7970	0.0089

This table reports the area under the ROC curve for Fama-French 10-industry classifications (2015). Note that if neither COMPUSTAT nor CRSP SIC codes were available, we assign these firms to a mixed industry classification. There are 327,835 firm quarter observations / 10,626 firms with a four digit SIC code classified in the Fama-French 10-industry definition (2015). Similar results are obtained, in untabulated analysis, by Fama-French 48-industry classifications (2015).

Table 11: Receiver Operating Characteristic by Delisting Codes

Panel A: Receiver Operating Characteristic (ROC): Area under the Curve (AUROC) by Delisting Codes (dlstcd) (accounting information)

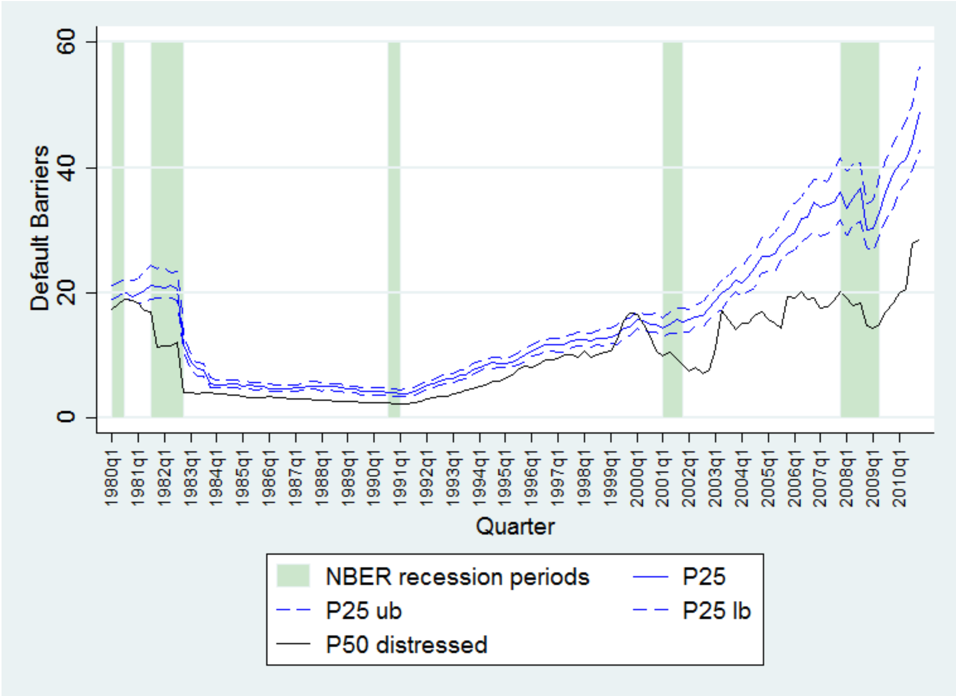
Model:	#		S-Prob		Z-Prob		O-Prob		Z-Prob^u		O-Prob^u		Z2-Prob^u	
Delisting Codes	solvent	delisted	AUROC	SE	AUROC	SE	AUROC	SE	AUROC	SE	AUROC	SE	AUROC	SE
400, 500, 550-585	288,552	41,722	0.8273	0.0010	0.7339	0.0014	0.7829	0.0012	0.7125	0.0014	0.8075	0.0011	0.6903	0.0015
400, 550-572, 574-585	289,416	40,858	0.8294	0.0010	0.7360	0.0015	0.7853	0.0012	0.7170	0.0014	0.8102	0.0011	0.6947	0.0015
400, 500, 550-572, 574-585	289,200	41,074	0.8296	0.0010	0.7361	0.0014	0.7854	0.0012	0.7151	0.0014	0.8090	0.0011	0.6927	0.0015

Panel B: Receiver Operating Characteristic (ROC): Area under the Curve (AUROC) by Delisting Codes (dlstcd) (market and accounting information)

Model:	#		S-Prob^m		EDF		C-Prob		C-Prob^u		Bhsh-Prob^u		Bhsh-Prob-DD^u		Beaver-Prob^u	
Delisting Codes	solvent	delisted	AUROC	SE	AUROC	SE	AUROC	SE	AUROC	SE	AUROC	SE	AUROC	SE	AUROC	SE
400, 500, 550-585	210,390	31,621	0.8487	0.0011	0.7706	0.0014	0.8252	0.0012	0.8413	0.0011	0.8347	0.0012	0.6595	0.0018	0.8468	0.0011
400, 550-572, 574-585	211,023	30,988	0.8502	0.0011	0.7718	0.0014	0.8283	0.0012	0.8425	0.0011	0.8357	0.0012	0.6616	0.0018	0.8476	0.0011
400, 500, 550-572, 574-585	210,848	31,163	0.8501	0.0011	0.7719	0.0014	0.8281	0.0012	0.8418	0.0011	0.8353	0.0012	0.6602	0.0018	0.8472	0.0011

This table reports the area under the ROC curve for multiple delisting code categories. In addition to the traditional definition of financial distress related delisting codes we examine three adjusted broader definitions (dlstcd=400, 500, 550-585), (dlstcd=400, 550-572, 574-585), (dlstcd=400, 500, 550-572, 574-585) to capture the delisting category (dlstcd=500) defined as "Issue stopped trading on exchange - reason unavailable" which shows financially distress risk developments as well. Finally, we exclude the delisting category (dlstcd=573) defined as "Delisted by current exchange - company request, deregistration (gone private)", as this category seems to not match financially distressed companies with reference to the poor performance across all models in this category and supplementary summary statistics for performance-related measures.

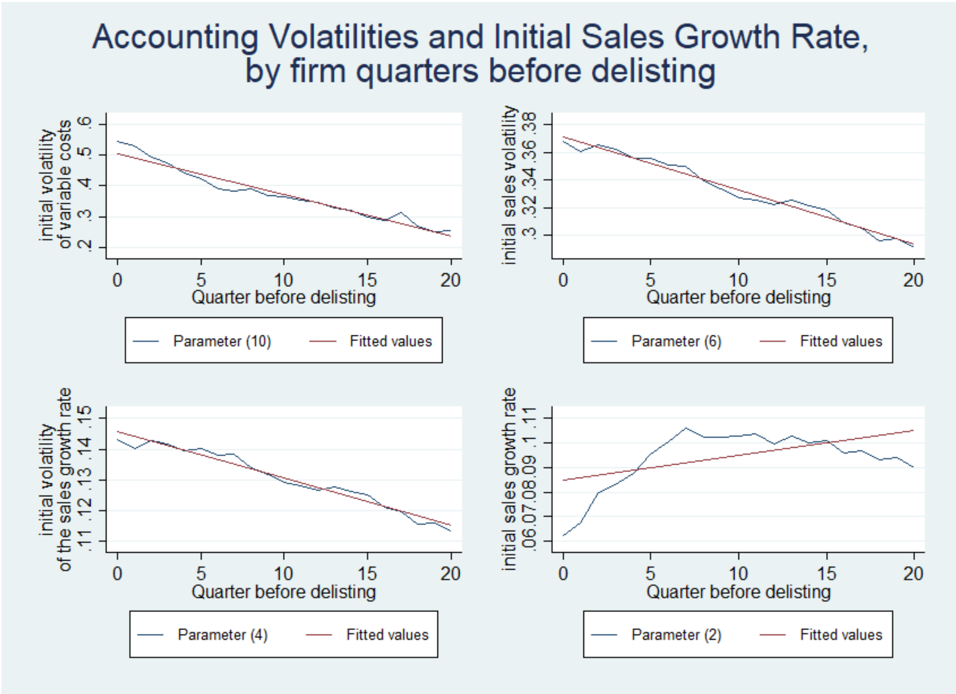
Figure 1: Distress Barrier Level Over Time



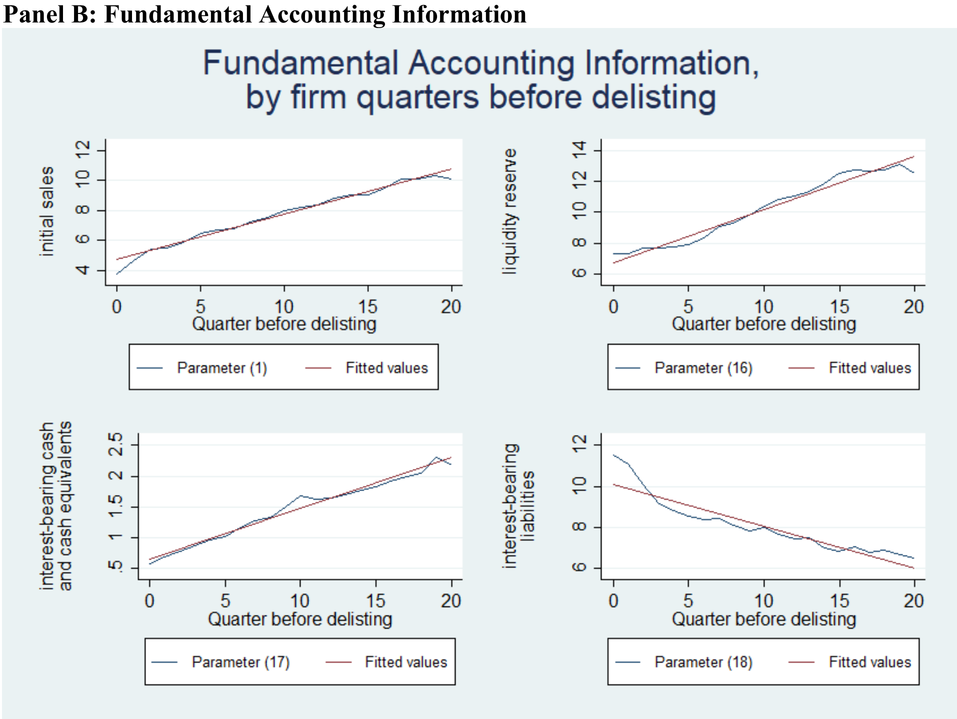
This figure plots the distress barrier *b* measured in million USD (blue solid line) estimated as 25-percentile of all quarterly liquidity reserve observations together with the lower/upper 95%-confidence boundaries (blue dashed lines) combined with the quarterly 50-percentile for financially distressed firms (black solid line) and the business-cycle status of the economy as announced by the NBER Business Cycle Dating Committee (see http://www.nber.org/cycles.html).

Figure 2: Comparison of Initial Parameters

Panel A: Accounting Volatilities and Initial Sales Growth Rate

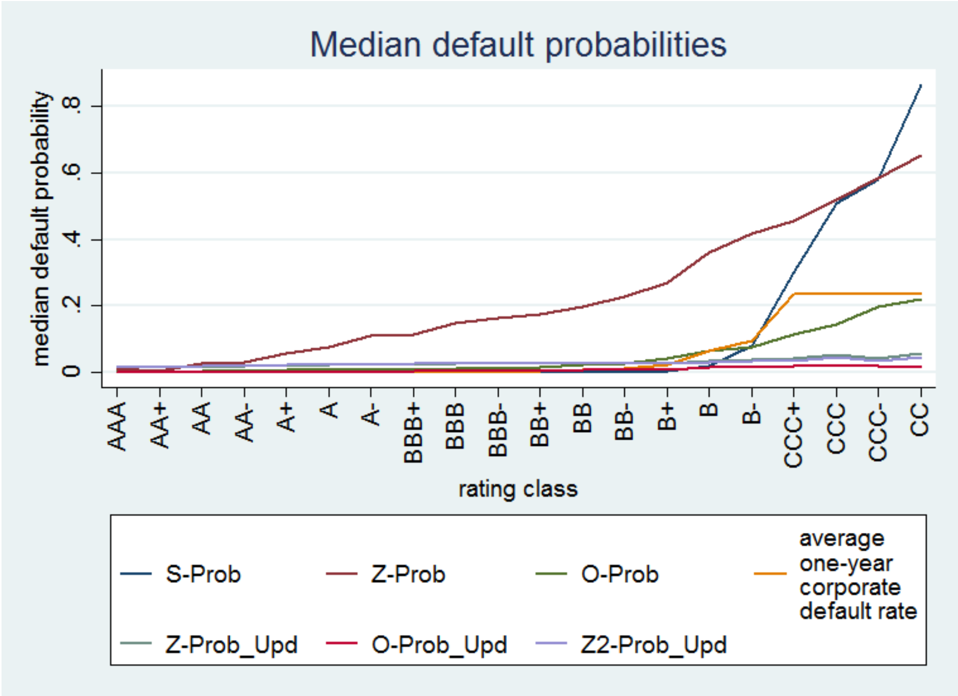


Panel B: Fundamental Accounting Information



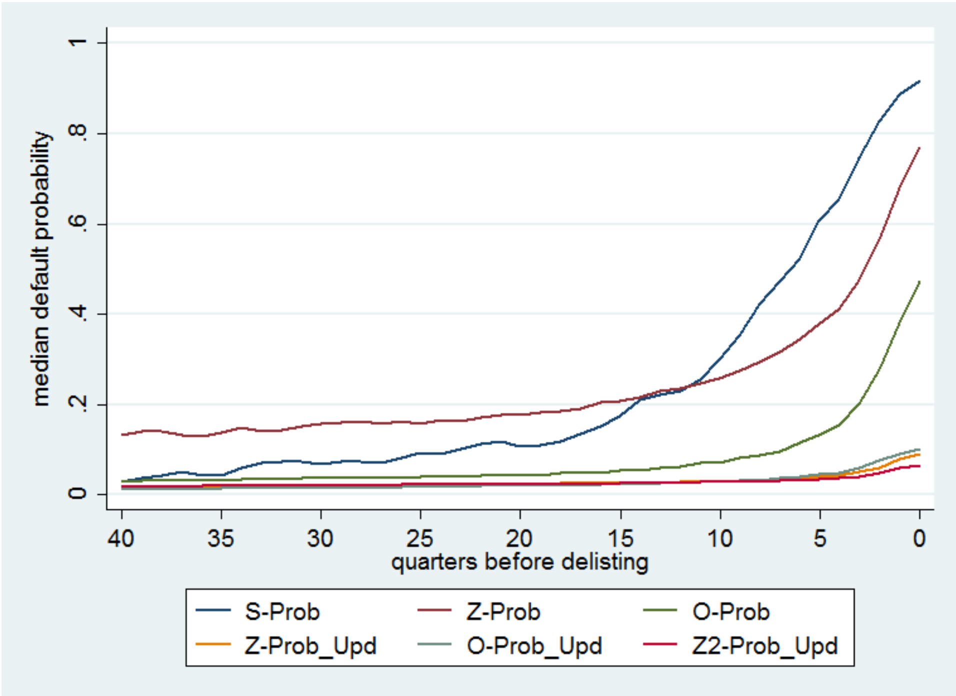
These figures show the mean accounting volatility measures (see No. 10, No. 4, No. 6 in table 3), mean initial sales growth rate (No. 2), median income statement and balance sheet positions (No. 1, No. 16, No. 17, No. 18) and along with the trend line for the quarters before a performance-related delisting (N=41,506 financially distressed firm quarters).

Figure 3: Median Estimated Default Probability per S&P Rating Class



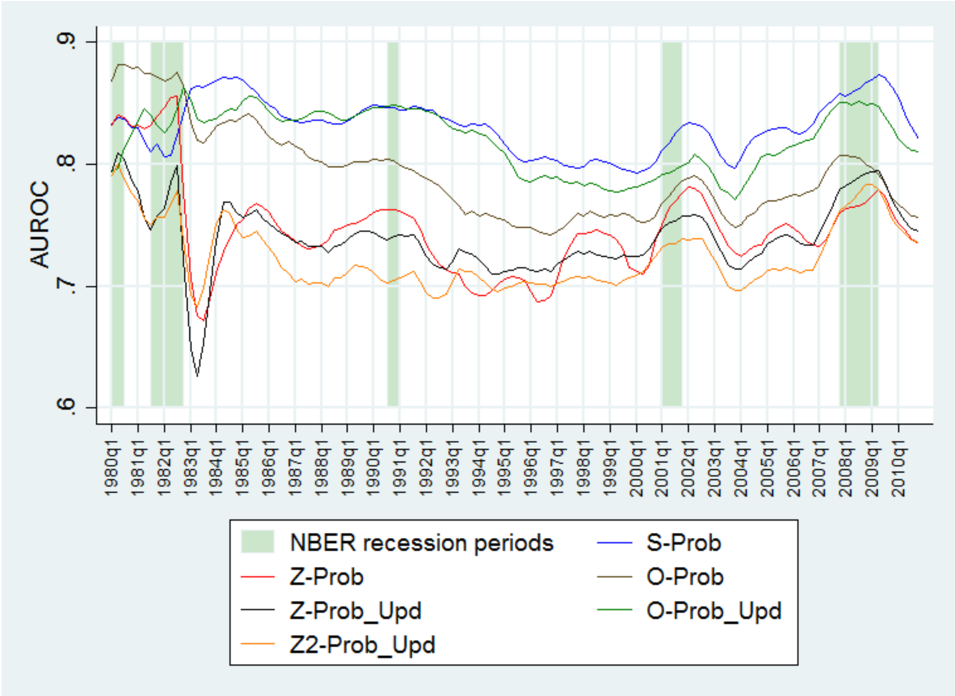
This figure shows the median estimated distress probability per Standard & Poor's rating class. RATING CLASS (=Standard & Poor's Credit Ratings, based on the COMPUSTAT item splticrm). The non-investment grade starts with ("BB"). For this analysis the sample size is reduced to 60,946 firm quarter observations / 2,243 firms that are covered by the major rating agency Standard & Poor's. The historic default rate as benchmark is from the 2013 Annual U.S. Corporate Default Study by Standard & Poor's Global Fixed Income Research.

Figure 4: Evolvement of Default Probabilities Before Delisting



This figure shows the evolvement of the median estimated default probabilities for the different models up to 40 quarters before delisting (from 1980Q1 to 2010Q4). The zero represents the quarterly end of an actual listing at one of the following stock exchanges (NYSE, AMEX, NASDAQ or NYSE ARCA) using CRSP delistings codes (dlstcd=400, 550-585).

Figure 5: ROC Analysis over Time



This figure shows the area under Receiver Operating Characteristic curve (AUROC) over time. The AUROC is calculated in a rolling window (one-year) following the nonparametric approach by DeLong et al. (1988) and Hanley and McNeil (1982) for each single quarter (1980Q1-2010Q4) using a five-year forecast horizon.

Appendix

Re-estimated accounting-based models (Appendix 1) and market-based financial distress prediction models (Appendix 2: expected default frequency *(EDF)* model, Appendix 3: *C-Score* model of Campbell et al. (2008), Appendix 4: *Bhsh-Score* and *Bhsh-DD-Score* model of Bharath and Shumway (2008), Appendix 5: Beaver et al. (2012) combined score model).

Results of the rolling regression to modify accounting-based volatilities (Appendix 6)

Appendix 1: Re-estimated Accounting-based Models

Table A.1: Summary statistics: Re-estimated Altman Z-Score, Ohlson-Score and Z2-Score Models

Panel A: Updated Z-Score Model (Z-Prob^u) (firm quarters from 1980Q1 – 2010Q4)					
	N	Original coeff.	Mean	Median	Std.dev.
Constant	124	-	(3.43)	(3.13)	0.71
X1=WC/TA	124	(1.20)	(1.76)	(1.85)	0.56
X2=RE/TA	124	(1.40)	(0.13)	(0.14)	0.10
X3=EBIT/TA	124	(3.30)	(10.38)	(9.41)	2.69
X4=MVE/TL	124	(0.60)	(0.12)	(0.02)	0.29
X5=SA/TA	124	(0.999)	0.09	0.02	0.45
Pseudo R^2	124	-	0.16	0.16	0.02

Panel B: Updated O-Score Model (O-Prob^u) (firm quarters from 1980Q1 – 2010Q4)					
	N	Original coeff.	Mean	Median	Std.dev.
Constant	124	(1.32)	0.27	0.55	1.67
X1=SIZE	124	(0.407)	(0.46)	(0.49)	0.11
X2=TLTA	124	6.03	2.08	1.88	0.65
X3=WCTA	124	(1.43)	(1.06)	(1.27)	0.68
X4=CLCA	124	0.0757	(0.08)	(0.10)	0.17
X5=NITA	124	(2.37)	(0.63)	(0.72)	0.38
X6=FULT	124	(1.83)	(0.57)	(0.42)	0.57
X7=INTWO	124	0.285	1.22	1.15	0.27
X8=OENEG	124	(1.72)	(1.51)	(1.46)	0.33
X9=CHIN	124	(0.521)	(0.32)	(0.28)	0.11
Pseudo R^2	124	-	0.22	0.22	0.03

(continued on next page)

(Table A.1 continued)

Panel C: Updated Z2-Score Model (accounting-based) (Z2-Prob^u) (firm quarters from 1980Q1 – 2010Q4)					
	N	Original coeff.	Mean	Median	Std.dev.
Constant	124	-	(3.54)	(3.21)	0.77
X1=WC/TA	124	(0.717)	(1.63)	(1.71)	0.52
X2=RE/TA	124	(0.847)	0.04	0.02	0.13
X3=EBIT/TA	124	(3.107)	(10.13)	(8.96)	2.60
X4=BVE/TL	124	(0.420)	(0.05)	0.00	0.17
X5=SA/TA	124	(0.998)	0.07	0.01	0.49
Pseudo R^2	124	-	0.13	0.13	0.02

Panel A reports summary statistics for the original and updated Z-Score model (estimated by a growing window logit regression model, starting in firm quarter 1976Q1). For example, we estimate the updated coefficients to calculate the Z-Score in 1980Q1 using firm quarter observations (X1-X5) from 1976Q1-1979Q3 and delisting information (CRSP delisting codes 400, 550-585) from 1979Q4 (out-of-sample-approach). We list the mean, median and standard deviation of the estimated coefficients by summarizing the main sample period of 1980Q1-2010Q4 (=124 firm quarters). We winsorize all measures at the 1st and 99th percentile to avoid outliers driving the results. The Z-Scores^u are transformed into probabilities using the standard logistic function (Z-Prob^u=1/(1+exp(-Z-Score^u)).

Panel B reports summary statistics for the updated O-Score model (estimated by a growing logit regression model, starting in firm quarter 1976Q1). As in Panel A, we ensure all measures are observable at that quarter over which the updated probabilities are calculated (out-of-sample-approach). We winsorize all measures at the 1st and 99th percentile to avoid outliers driving the results. Consistent with Begley et al. (1996) and Hillegeist et al. (2004), we find that the coefficients are not stable across different time periods. The O-Scores are transformed into probabilities using the standard logistic function (O-Prob^u=1/(1+exp(-O-Score^u)).

Panel C reports summary statistics for the updated accounting-based (or private firm) Z2-Score model (estimated by a growing logit regression model, starting in firm quarter 1976Q1). Again, we winsorize all measures at the 1st and 99th percentile to avoid outliers driving the results and ensure all measures are observable at that quarter over which the updated probabilities are calculated (out-of-sample-approach).

WC/TA = working capital/total assets; RE/TA = retained earnings/total assets; EBIT/TA = earnings before interest and taxes/total assets; MVE/TL = market capitalization/total liabilities; SA/TA = sales/total assets; SIZE = ln(total assets/ lagged GNP level (with base year 1968)); TLTA= total liabilities/total assets; CLCA= current liabilities/current assets; NITA= net income/total assets; FULT=pretax income/total liabilities; INTWO = 1 if the net income in the last two quarter were negative and zero otherwise; OENEG= 1 if total liabilities > total assets; CHIN=(net income$_t$ – net income$_{t-1}$)/(|net income$_t$|+|net income$_{t-1}$|); BVE/TL= stockholders equity (+ deferred taxes and investment tax credit) - preferred stocks/total liabilities (note: in the case of missing data we use the common equity plus the carrying value of preferred stock. If neither shareholder equity nor common equity is available, we calculate the numerator (BVE) as total assets minus total liabilities); Pseudo-R^2= defined by McFadden's (1974)-Pseudo-R^2: $(1-L_1/L_0)$.

Appendix 2: Expected Default Frequency (EDF) - Measure

The contingent claims-based framework is a widely used operationalization of a market-based prediction model that requires (daily) market and accounting observations of the following input parameters: (1) the total firm value of assets (V=iteratively inferred), (2) the related volatility of the firm assets (σ_V=iteratively inferred), (3) the face value of total debt maturing at time T (B=debt in current liabilities plus one-half of the long-term interest bearing debt), (4) the value of equity (E=market value of equity from CRSP), (5) the expected return on assets (μ=iteratively inferred), (6) the prediction horizon T, and (7) the risk-free rate (r=1 year US Treasury Bill Rate). The underlying assumptions of the contingent claims framework are as follows: (i) the total firm value of assets V follows a geometric Brownian motion and (ii) the total debt, i.e., the total claims of a firm, maturing at time T.[19] To receive estimates for the firm value of assets V and volatility of assets σ_V, we operationalize the sequential-iterations algorithm as described by Crosbie and Bohn (2003), Vassalou and Xing (2004), Bharath and Shumway (2008) and more recently in Jessen and Lando (2015) with a maximum of 15 iterations. The value of equity, viewed as a European call option is obtained from the following:

$$E(V;T) = VN(d_1) - Be^{-rT}N(d_2) \tag{21}$$

$$d_1 = \frac{\ln(V/B) + (r + 0.5\sigma_V^2)T}{\sigma_V \sqrt{T}} \tag{22}$$

$$d_2 = d_1 - \sigma_v \sqrt{T} = \frac{\ln(V/B) + (r - 0.5\sigma_V^2)T}{\sigma_V \sqrt{T}} \tag{23}$$

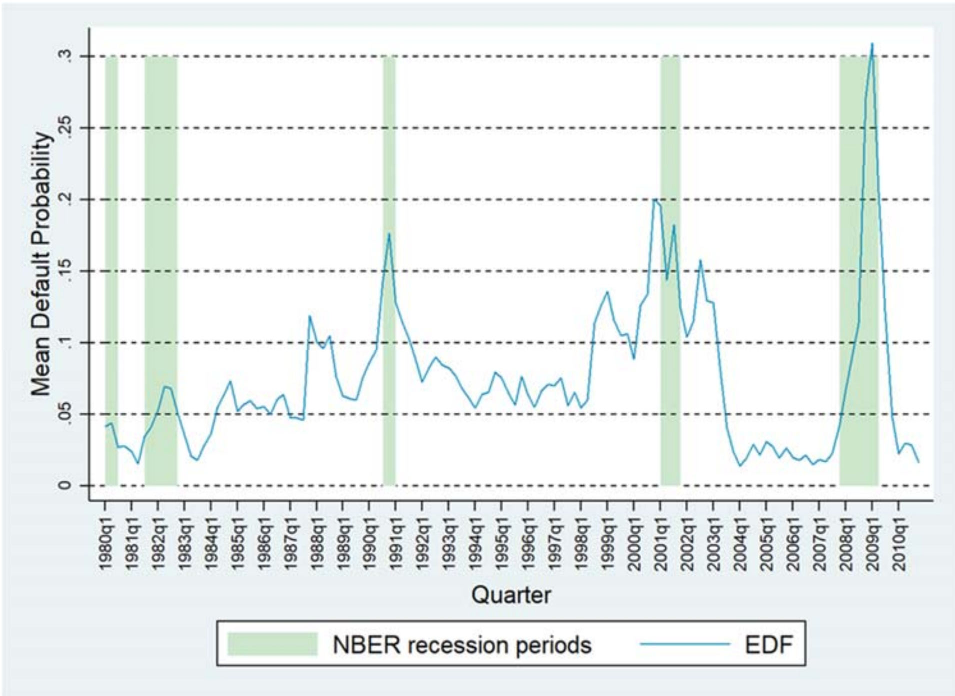
where we require, similar to Bharath and Shumway (2008), a forecast horizon of T=1 year and at least 60 trading days over the previous 12 months. Finally, the probability of default EDF is given by the following:

$$\begin{aligned} EDF &= \mathrm{P}(\mathrm{V}_T \leq B) = 1 - N(d_2) = N(-\mathrm{d}_2) \\ &= N\left(-\frac{\ln(V/B) + (\mu - 0.5\sigma_V^2)T}{\sigma_V \sqrt{T}}\right) \end{aligned} \tag{24}$$

[19] To be precise, the theoretical underpinning of the Black-Scholes model requires additional assumptions, for example lognormal distributed returns, constant volatilities and other related market frictions (Agarwal and Taffler 2008).

where N(·) is the cumulative standard normal distribution function.[20] We winsorize all model parameters at the 1st and 99th percentile to avoid outliers driving the results.

Figure A.2: EDF probability (time-series)



This table plots cross-sectional mean default probabilities with reference to the EDF model following the algorithm in Bharath and Shumway (2008) over quarterly periods 1980Q1 to 2010Q4.

[20] Additionally, we compare our estimated *EDF*-measure with the aggregated default likelihood indicator presented in Vassalou and Xing (2004). We thank the authors Maria Vassalou and Yuhang Xing for making their results available at link: http://maria-vassalou.com/research/data. Based on the merged 146,189 firm quarter observations with non-missing information from 1980Q-1999Q4, we compare the AUROC of both measures. The findings indicate, as expected, that the performance of implied default probability following Bharath and Shumway (2008) and the default probability by Vassalou and Xing (2004) are not significantly different from each other ($AUROC_{Bharath/Shumway}$ = 0.7650 vs. $AUROC_{Vassalou/Xing}$ = 0.7651). The test is based on an indicator variable for financial distress that equals one if a delisting (indicated by CRSP delisting codes 400, 550-585) occurs within the next 20 quarters (i.e., a delisting firm quarter is determined if the firm experience a delisting within the next 20 quarters as defined in Table 1).

Table A.2: Results for the re-estimated EDF model of Bharath and Shumway (2008)

Univariate summary statistics (N=1,690,677 firm month from 1980|01 to 2010|12)						
No.	Variable	Mean	Median	Std.dev.	1%	99%
1	V	1184.38	133.34	3300.54	2.11	22775.32
2	σ_V (%)	51.64	42.27	34.33	7.77	187.09
3	B	251.50	16.54	788.45	0.02	5617.71
4	E	926.99	95.41	2712.39	1.36	19320.34
5	μ (%)	3.68	5.37	57.70	-196.96	180.63
6	r (%)	-0.01	-0.40	3.65	-11.46	15.87
7	edf (%)	8.42	0.00	20.39	0.00	93.23

This table reports summary statistics for the re-estimated EDF model (estimated following the algorithm in Bharath and Shumway (2008)). V is the firm value of assets, σ_V equals the volatility of the firm assets, B is defined as current liabilities plus one-half of the long-term interest bearing debt (COMPUSTAT #45 (dlcq) and #51 (dlttq)), E is the market value of equity in mio. USD taken from CRSP, μ is the expected return on assets, r is the 1-month firm's equity return over the 1-month Treasury-bill risk-free rate, and edf equals the estimated probability of default following Eq. 24. The table lists the mean, median, standard deviation and the lower and upper percentiles of the main parameters by summarizing the sample period of 1980|01-2010|12 (=372 firm months) with N=1,690,677 firm-months. We winsorize all input parameters at the 1st and 99th percentile and report the iterative inferred model results. Results are in line with prior studies.

Appendix 3: Re-estimated Campbell et al. (2008) - Model

Table A.3: Results for the original and re-estimated C-Score Model No. 2 of Campbell et al. (2008)

*Updated C-Score Coefficients (*C-Prob^{u}*) (firm month from 1980|01 – 2010|12)*					
	N	Original coeff.	Mean	Median	Std.dev.
Constant	372	(9.08)	(15.75)	(15.21)	1.59
X1=NIMTAAVG	372	(29.67)	(9.38)	(8.03)	3.77
X2=TLMTA	372	3.36	1.26	1.14	0.64
X3=EXRETAVG	372	(7.35)	(1.71)	(2.15)	1.70
X4=SIGMA	372	1.48	0.57	0.48	0.41
X5=RSIZE	372	0.082	(0.66)	(0.67)	0.06
X6=CASHMTA	372	(2.40)	(1.53)	(1.60)	0.91
X7=MB	372	0.054	0.07	0.07	0.01
X8=PRICE	372	(0.937)	(0.32)	(0.41)	0.19
Pseudo R^2	372	0.312	0.26	0.27	0.02

This table reports summary statistics for the original and re-estimated *C-Score* model (estimated by a growing window proportional hazard regression model, starting in firm month 1976|01). For example, we estimate the updated coefficients to calculate the *C-Score* in January of 1980 using firm month observations (X1-X8) as defined by the Appendix of Campbell et al. (2008) from January 1976-November 1979 and monthly delisting information (CRSP delisting codes 400, 550-585) from December 1979 (out-of-sample-approach). We list the mean, median and standard deviation of the coefficients by summarizing the main sample period of 1980|01-2010|12 (=372 firm months). Consistent with the alternative failure model (Model 1) of Campbell et al. (2008) and the re-estimated version of Bauer and Agarwal (2014) the coefficients have the expected signs. Intuitive and contrary to the specifications in Campbell et al. (2008) failure model no. 2 the coefficient on RSIZE is strictly negative in the re-estimated version implying that firms with relative more market capitalization are less likely to fail. Bauer and Agarwal (2014) confirm these results in their re-estimated version. We winsorize all measures at the 1st and 99th percentile to avoid outliers driving the results. The C-Scores are transformed into probabilities using the standard logistic function (C-Prob^{u}=1/(1+exp(-C-Score^{u})). Pseudo R^2= defined by McFadden's (1974)-Pseudo R^2: $(1-L_1/L_0)$.

Appendix 4: Re-estimated Bharath and Shumway (2008) - Models

Table A.4: Results of the re-estimated Bhsh-Score/BhSh-DD-Score Models No. 7/2 in Bharath and Shumway (2008)

Panel A: Updated BhSh-Score Coefficients (bhsh-prob^{u}) (firm month from 1980|01 – 2010|12)					
	N	Original coeff*	Mean	Median	Std.dev.
Constant	372	-	(4.39)	(4.00)	(4.39)
X1=naïve DD	372	1.526	1.65	1.40	1.65
X2=log(EQUITY)	372	(0.255)	(0.82)	(0.83)	(0.82)
X3=log(DEBT)	372	0.269	0.04	0.03	0.04
X4=$1/\sigma_E$	372	(0.518)	(0.39)	(0.40)	(0.39)
X5=EXCESSRETURN	372	(0.834)	(0.28)	(0.43)	(0.28)
X6=NI/TA	372	(0.044)	(0.06)	(0.06)	(0.06)
Pseudo R^2	372	-	0.27	0.28	0.27

Panel B: Updated BhSh-DD-Score Coefficients (bhsh-dd^{u}) (firm month from 1980|01 – 2010|12)					
	N	Original coeff*	Mean	Median	Std.dev.
Constant	372	-	(7.18)	(6.73)	(7.18)
X1=naïve DD	372	4.011	3.93	3.90	3.93
Pseudo R^2	372	-	0.12	0.11	0.12

This table reports summary statistics for the original and re-estimated *BhSh-Score* model. BhSh^{u} / BhSh-DD^{u} are defined as in Bharath and Shumway (2008), model 7 / model 2 using the naïve alternative distance-to-default measure (re-estimated by a growing window proportional hazard regression model, starting with month 1976|01). We list the mean, median and standard deviation of the coefficients by summarizing the main sample period of 1980|01-2010|12 (=372 firm months). Additionally, all measures are winsorized at the 1st and 99th percentile to avoid outliers driving the results. The *BhSh-scores* and *BhSh-DD-scores* are transformed into probabilities using the standard logistic function (e.g., BhSh-prob^{u}=1/(1+exp(-BhSh-Score^{u})). Pseudo R^2= defined by McFadden's (1974)-Pseudo R^2: $(1-L_1/L_0)$. naïve DD= naïve alternative distance-to-default probability as defined in Bharath and Shumway (2008, p. 1348); ln(EQUITY) and ln(DEBT) are the natural logarithms of the market value of equity and the current plus one-half long-term value of debt; $1/\sigma_E$ = inverse past 12-month equity volatility as measured by daily price data from CRSP; EXCESSRETURN = cumulative past 12-month log excess return relative to the S&P500; NI/TA = net income/total assets.

*Carefully note that Bharath and Shumway (2008) do not report any constants given they directly operationalize the Cox proportional hazard technique with time-varying covariates.

Appendix 5: Re-estimated Correia et al. (2012) - Model

Table A.5: Results of the re-estimated Beaver-Score as defined by Correia et al. (2012)

*Updated Beaver-Score Coefficients (*Beaver-prob^{u}*) (firm month from 1980|01 – 2010|12)*				
	N	Mean	Median	Std.dev.
Constant	372	(20.81)	(21.06)	0.92
X1=NROAI	372	0.84	0.82	0.17
X2=ROA	372	(1.27)	(1.45)	0.99
X3=TLTA	372	3.06	2.79	0.50
X4=EBITTL	372	(0.20)	(0.15)	0.22
X5=RETURNS	372	(0.64)	(0.54)	0.24
X6=σ_E	372	0.50	0.42	0.38
X7=LRSIZE	372	(0.93)	(0.98)	0.11
X8=NROAI*ROA	372	0.30	0.29	0.55
X9= NROAI*TLTA	372	(0.14)	(0.15)	0.10
X10= NROAI*EBITTL	372	(0.08)	(0.00)	0.52
Pseudo R^2	372	0.30	0.32	0.04

This table reports summary statistics for the re-estimated *Beaver-Score* (i.e., Beaver et al. (2012) combined score) model (estimated by a growing window logit regression model, starting in firm month 1976|01). For example, we estimate the updated coefficients to calculate the *Beaver-Score* in January of 1980 using firm month observations (X1-X10) as defined by Correia et al. (2012) from January 1976-November 1979 and monthly delisting information (CRSP delisting codes 400, 550-585) from December 1979 (out-of-sample-approach). We list the mean, median and standard deviation of the coefficients by summarizing the main sample period of 1980|01-2010|12 (=372 firm months). We winsorize all measures at the 1st and 99th percentile to avoid outliers driving the results. The results reveal that all coefficients have the expected signs with reference to Table 3 in Beaver et al. (2012), and, by comparison, the combined Beaver et al. (2012) model experiences the highest explanatory power (median R^2 = 0.32). The *Beaver-Scores* are transformed into probabilities using the standard logistic function (Beaver-Prob^{u}=1/(1+exp(-Beaver-Score^{u})). Pseudo R^2= defined by McFadden's (1974)-Pseudo R^2: (1-L_1/L_0).

Appendix 6: Estimation of Modified Accounting-based Volatilities

Table A.6: Results of the rolling regression models for modified accounting-based volatilities

Coefficients of a rolling cross-sectional regression model with constraints (firm quarter from 1980|01 – 2010|12)

Model 1: ($\sigma_{S,t} = \alpha + \beta_1 \sigma_{S,t-1} + \beta_2 \sigma_{mar,t-1}$)

	N	Mean	Median	Std.dev.
Coeff β_1	124	0.98	0.99	0.01
Coeff β_2	124	0.02	0.01	0.01

Model 2: ($\sigma_{g,t} = \alpha + \beta_1 \sigma_{g,t-1} + \beta_2 \sigma_{mar,t-1}$)

	N	Mean	Median	Std.dev.
Coeff β_1	124	0.99	0.99	0.01
Coeff β_2	124	0.01	0.01	0.01

Model 3: ($\sigma_{c,t} = \alpha + \beta_1 \sigma_{c,t-1} + \beta_2 \sigma_{mar,t-1}$)

	N	Mean	Median	Std.dev.
Coeff β_1	124	0.95	0.95	0.02
Coeff β_2	124	0.05	0.05	0.02

This table reports summary statistics for coefficients required to estimate the adjusted accounting-volatilities. $\sigma_{S,t}$ is the sales specific volatility, $\sigma_{g,t}$ is the sales growth rate volatility, $\sigma_{c,t}$ is the cost volatility, and $\sigma_{mar,t-1}$ is the three-month rolling standard deviation of the firm's stock returns centered around zero using daily observations from CRSP.